FLEXÃO INERCIAL DE UMA GUIA DE ONDAS COM PAREDES FINAS DE SECÇÃO TRANSVERSAL RETANGULAR: FLEXÃO DA PAREDE LATERAL EM SEU PLANO

INERTIAL BENDING OF A THIN-WALLED WAVEGUIDE WITH RECTANGULAR CROSS-SECTION: BENDING OF LATERAL WALL IN ITS PLANE

ИНЕРЦИОННЫЙ ИЗГИБ ТОНКОСТЕННОГО ВОЛНОВОДА ПРЯМОУГОЛЬНОГО ПОПЕРЕЧНОГО СЕЧЕНИЯ: ИЗГИБ БОКОВОЙ СТЕНКИ В СВОЕЙ ПЛОСКОСТИ

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RESUMO

Este artigo propõe um método para modelar barras de paredes finas com base na teoria de placas e cascas. Com base nas disposições da teoria de placas e cascas e no método semi-inverso de Saint-Venant, é proposta uma solução analítica parcial de um sistema de equações diferenciais não-lineares com derivadas parciais para calcular o estado tensão-deformação de placas laterais das secções retas das paredes finas de guias de onda de secção transversal retangular sobre o efeito da carga inercial. A novidade deste artigo consiste no fato de que o estudo realizado comprovou a razão do comportamento de um guia de onda de secção transversal retangular. A novidade deste artigo consiste no fato de que o estudo realizado comprovou a razão do comportamento de um guia de onda de seção transversal retangular. Para alcançar os resultados desejados, foram utilizados métodos de cálculo baseados nos trabalhos de Vlasov, Bychkov e Rzhanitsyn. Os resultados obtidos mostraram a correção da metodologia e revelaram a necessidade de esclarecer a fórmula de Zhuravsky no cálculo das tensões tangenciais em barras de paredes finas.

Palavras-chave: *barra de paredes finas, flexão, método semi-inverso de Saint-Venant, solução analítica, função de Airy*.

ABSTRACT

Based on the theory of plates and shells, a method of modeling thin-walled beams is proposed. Using the tenets of the theory of plates and shells, Papkovich–Neuber solution, as well as the Saint Venant semi-inverse method, partial analytical solution of a system of nonlinear partial differential equations, is applied to calculate inertial stress action on the deflected mode of lateral plates of thin-walled straight parts of waveguides with a rectangular cross-section. The novelty of this article is in the fact that the conducted research proved the cause of the behavior of a waveguide with rectangular cross-section. To achieve the desired results, calculation methods were used, based on the works of Vlasov, Bychkov and Rzhanitsyn. The obtained results showed the correctness of this technique and revealed the necessity to specify the Zhuravsky formula when calculating shear stresses in thin-walled beams by Euler–Bernoulli beam theory.

Keywords: *thin-walled beam, bending, Papkovich-Neuber solution, Saint Venant semi-inverse method, analytical solution, Airy function*.

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АННОТАЦИЯ

В данной работе предлагается способ моделирования тонкостенных стержней на основе теории пластин и оболочек. На основе положений теории пластин и оболочек и полуобратного метода Сен-Венана предлагается частное аналитическое решение системы нелинейных дифференциальных уравнений в частных производных для расчета напряжённо-деформированного состояния боковых пластинок тонкостенных прямых участков волноводов прямоугольного поперечного сечения на действие инерционного нагружения. Новизна данной статьи заключается в том, что проведенное исследование доказало причину поведения волновода прямоугольного поперечного сечения. Для достижения нужных результатов применялись методы расчета, на основе работ Власова, Бычкова и Ржаницына. Полученные результаты показали корректность методики и выявили необходимость уточнения формулы Журавского при расчете касательных напряжений в тонкостенных стержнях.

Ключевые слова: *тонкостенный стержень, изгиб, полуобратный метод Сен-Венана, аналитическое решение, функция Эри, напряжение*.

INTRODUCTION

An analysis of deflected mode (DM) for extensive thin-walled constructions with non-axis-symmetrical cross-section is very complicated and ambiguous because such constructions are intermediate between the Euler-Bernoulli beam theory and theory of shells (Barbosa *et al.*, 2017; Volmir, 1956). Verification of validity criteria of both the above theories for thin-walled beams gives a positive reply; however, the results of calculations obtained with them often differ considerably.

Application of the Euler–Bernoulli beam theory (Feodosiev, 1999; Beer *et al.*, 2001) leads to excessive simplification of calculated models of thin-walled constructions in the form of 1D longitudinal axes having equivalent geometric and inertial characteristics, loads and fixation methods. This makes it possible to estimate DM of extended thin-walled constructions with a non-axis-symmetrical cross-section as a whole, without taking into account the features of thin-walled elements behavior under load (e.g., a variation of cross-section geometry or even loss of local stability) (Sokolov, 2002; Galanin, 2012; Formalev *et al.*, 2016). To remove such shortcomings, a variety of special-purpose approaches to the calculation of thin-walled beams have been developed since the middle of the twentieth century.

The simplest solution used when calculating extensive thin-walled beams with non-axis-symmetrical cross-section is the application of the Euler–Bernoulli beam theory added with adjustment coefficients (Agamirov, 2003). However, the obtained results highly depend on the choice of values of such adjustment coefficients (Formalev *et al.*, 2017). The reference sources have a limited number of the coefficient values (only for a combination of certain forms and sizes of beam cross-section); in other cases, their choice is rather difficult.

The calculation methods based on the fundamental works by Vlasov (1963), Rzhanitsyn (1982) and Bychkov (1962) are the development of the theory of thin-walled beams with a non-axis-symmetrical cross-section. One of the achievements of the above works is an estimation and taking into account depletion of thin-walled beam cross-section in the case of high shear stresses (e.g., at bending torsion or joint action of bending and torsion). However, the corrected basic dependences of beams theory obtained in (Bychkov, 1962; Vlasov, 1963; Rzhanitsyn, 1982) are very complicated and non-universal; in addition, they cannot achieve the required calculation accuracy in local areas. To simplify calculation of thin-walled beams, Slivker proposed the so-called half-shear theory (Slivker, 2005) in which a state of a thin-walled beam is described within the Bernoulli-Euler shear-free law. The grave drawback of all above-mentioned methods of thin-walled beams calculation is that they are based on the Euler–Bernoulli beam theory. This cannot completely remove the above-mentioned drawbacks inherent to it.

We propose an approach in which any thin-walled spatial construction is a system of separate elements (plates, shells) joined together (Novozhilov *et al.*, 2010; Timoshenko and Woinowsky-Krieger, 1959; Timoshenko, 2009). When describing DM of such a construction, a separate local subsystem of differential equations is built for each of its individual elements. As a result, we have a general global system of equations. The complexity of the proposed method is that it is impossible to get a general analytical solution for such a system of differential equations (even for thin-walled beams with a cross-section of simple form). However, the obtained solutions for simple cases of loading are quite simple and can serve as a standard when comparing.

For example, the proposed approach was applied in (Silchenko *et al.*, 2015; Silchenko *et al.*, 2010) to determine corrected stressed state of straight part of a thin-walled waveguide with a rectangular cross-section. The waveguide construction was modeled with four plates (that formed its rectangular cross-section) and the corresponding systems of differential equations. The obtained analytical solution made it possible to correct the Navier equation and determine the values of shear stresses at pure bending, while the performed verification in Ansys package revealed some features in the application of finite elements of different types.

In this work, we consider inertial loading of a waveguide at which it is bending in the plane of minor rigidity. Inertial loading is an inherent type of loading for all static and moving objects under investigation (except the cases of weightlessness). Inertial loading of elements and constructions occurs as a result of the action of both their own weight and acceleration induced by different factors (terrestrial attraction, increasing motion, mechanical oscillations, etc.).

As a rule, taking account of inertial loading substantially complicates the obtained analytical dependences when calculating DM of an object, so this is often neglected. This is favored by the fact that the values of components of induced by workload stresses and deformations in constructions usually exceed (by an order of magnitude and more) the corresponding components due to their weight. However, in some cases (e.g., aerospace vehicles) inertial loading plays a decisive role. In such cases application of simplified calculation methods may result in large calculation errors, especially in local sites with various (geometric and physical) construction inhomogeneities.

Our approach will make it possible to get more correct and exact solution which is of importance for such critical component as a waveguide, as well as to extend realization of the problem of calculating lengthy thin-walled constructions with a non-axis-symmetrical cross-section.

It should be noted that the complete solution for the problem of waveguide DM determination using the theory proposed by us is very complicated. So this work considers the first step of solving: DM determination for lateral walls at inertial bending with allowance made for interaction with the rest of waveguide construction.

METHODOLOGY

2.1. Statement of the problem

Let us consider a design diagram of a waveguide straight section as a set of four plates fixed by a hinge (Figure 1). The waveguide experiences acceleration a_Y that results in its bending in the XY plane of the global coordinate system of the construction. Let the known bending moments M_{Z0} act along the waveguide edges to allow for waveguide interaction with other construction elements.

According to our approach (Silchenko *et al.*, 2015), the waveguide construction is presented as a set of four plates whose static, dynamic and temperature states are described by the following system of nonlinear differential equations (Equation 1), where *i* = 1, 2, 3 and 4 are the plate numbers; Equation (2):

$\alpha_i \beta_i z_i$ is the local coordinate system of the *i*-th plate;

$\omega_i = \omega_i(\alpha_i, \beta_i)$ is the bending function for the *i*-th plate;

$\varphi_i = \varphi_i(\alpha_i, \beta_i)$ is the Airy stress function for stresses in the *i*-th plate;

$q_{\alpha i}, q_{\beta i}, q_{Zi}$ are the components of surface load for the *i*-th plate;

E is Young's modulus; α is the coefficient of thermal expansion; τ is time;

t is the plate thickness; ρ is the plate material density;

$T_{0i}(\alpha_i, \beta_i)$ and $T_i(\alpha_i, \beta_i)$ are the initial and current temperature field of the *i*-th plate, respectively;

D_i is the bending stiffness of the *i*-th plate at the straight part.

Each group of the *i*-th equations in the system Eqs. (1) for the straight part describes DM of such *i*-th plate at whose contour all boundary conditions are to be set. According to (Gorshkov, 2002; Neuber, 1934), the boundary conditions at plate lateral sides are Equation (3). Let us consider an example of obtaining analytical solution when determining waveguide DM at inertial bending relative to *Z* axis in the global coordinate system at the plane of minimal rigidity when maximal stresses appear. At an isothermal loading of the straight part, the temperature component in the first equation of system Eqs (1) is zero: $E\alpha \cdot \nabla^2 [T_i(\alpha_i, \beta_i) - T_{0i}(\alpha_i, \beta_i)] = 0$, and there is no surface load at static loading: $q_{\alpha i} = q_{\beta i} = q_{Zi} = 0$.

As applied to the above special case of bending, derivation of the analytical solution of system Equation (1) is complicated by interrelation between separate differential equations as well as nonlinearity of the system as a whole. At inertial bending of the waveguide straight section (Figure 1a), free deformation of its constituent plates in their transverse directions z_i is assumed. Therefore, a flat stressed state appears in the plates, and then $\sigma_{Zi} = 0$.

The main feature of waveguide qualitative operation is that both sizes and form of the inner cross-section canal have to remain practically constant in the course of waveguide exploitation (permissible changes of their values must not exceed 0.1%). As applied to the calculation procedure for the waveguide straight section, these deformation features of plates operation are obtained on condition that Equation (4).

In geometrical terms, the condition Equation (3) is equivalent to the demand that the lines forming a cross-section of the waveguide straight section should remain direct under load. Substitution of condition Equations (3) into system Equations (1) and boundary condition Equation (2) lead to their substantial simplification; now they are as follows Equations (5, 6).

The conducted review of the mathematical literature showed that at present there are no analytical solutions for systems Equations (4, 5). A partial solution was obtained for the first time in (Silchenko *et al.*, 2015; Kudryavtsev *et al.*, 2017) when determining corrected stressed state for a straight section of waveguide thin-walled construction with rectangular cross-section at pure bending. Let us apply a similar approach to get an analytical solution to the problem of inertial bending of lateral plates in their planes as part of waveguide construction.

2.2. Solving the problem

To solve the considered problem of inertial bending Equations (4, 5), let us apply the Papkovich–Neuber solution with Saint Venant semi-inverse method (Neuber, 1934; Papkovish, 1932; Papkovish, 1939; Parton, 1981; Timoshenko, 1976; Aleksandrov, 1990) and immediate determination of stresses (the Airy function $\varphi_i(\alpha_i, \beta_i)$) and shiftings (bendings $\omega_i(\alpha_i, \beta_i)$). As a result, we get partial solutions for separate plates. Association of them is an analytical solution for the considered problem of bending of the straight section of the waveguide as a whole.

In a case of straight section bending, solution for the functions $\varphi_i(\alpha_i, \beta_i)$ and $\omega_i(\alpha_i, \beta_i)$ can be built based on the DM features for each of its *i*-th plate (Figure 1a):

– the lateral plates 2 and 4 experience lateral bending in their planes;

– the plates 1 and 3 experience stretching and compression, respectively, together with bending in a curve formed by deformed edges of the lateral plates 2 and 4.

As a result of the proposed approach, the waveguide DM will be mostly determined by the state of the lateral walls 2 and 4 for which it is necessary to get an analytical solution as the Airy functions $\varphi_i(\alpha_i, \beta_i)$ in the system Equation (1).

2.3. Determination of lateral plate stresses

Let us consider the lateral plate 2 that is bending under its own weight as a part of the waveguide construction (Figure 2). Besides its own specific weight γ, a shear stress $\tau_2^{'}$ at the junctures with the plates 1 and 3 as well as distributed stress from pressure σ_R will act on its upper and lower sides. A design diagram for the dedicated plate 2 made with allowance for its interaction with the rest rejected part of waveguide construction is shown in Figure 2.

Taking into consideration all the features of loading, let us assume that the Airy function $\varphi_i(\alpha_i, \beta_i)$ for plate 2 is a fifth-degree polynomial (Papkovish, 1939; Aleksandrov, 1990) (Equation (7). In this case, the expressions for stresses that determine DM of the plate 2 are based on the known dependences of the Airy equations; with allowance for the inertial forces, they are Equation (8, 9, 10).

Let us obtain coefficients in the polynomial Equation (7) by substituting this expression in the boundary conditions at the sides of plate 2. Interaction of the lateral plate 2 with the upper and lower plates 1 and 3 occur at its upper and lower sides where the shear stresses are Equation (11). The second boundary condition determines the values of normal stresses σ_R due to the pressure of the plates 1 and 3 on the plate 2 at their junctures (Equation 12). The third boundary condition determines the value of the bending moment M_{Z0} along the edges of the waveguide straight section which is taken as integral value distribution of normal stresses $\sigma_{\alpha 2}$ along the height of cross-section for the lateral plate 2. For the adopted design diagram of the waveguide (Figure 1), this boundary condition is Equation (13).

After substituting Equation (7) in the conditions Equations (8-10), we get the required coefficients in the Airy function (Equations 14-17). Equations (7) with coefficients Equation (11) completely determine the stressed state in the lateral plate 2 of the waveguide at its inertial loading. Owing to the symmetry of the waveguide construction geometry as well as conditions of its fixation and loading, the solution Equations (7, 11) will be also true for plate 4.

2.4. Determination of lateral plate deformations

We determine a deformed state of the plate 2 based on the Airy function using the Papkovich procedure (Neuber, 1934; Papkovish, 1932; Papkovish, 1939). Let us present the Airy function as Equation (18), where Equation (19). According to (Neuber, 1934; Papkovish, 1932; Papkovish, 1939), the plate shiftings are as follows: Equation (20, 21), where Equation (22). After substitution and cancellations, we get longitudinal shiftings as Equation (23), where Equations (24-26). The transversal shiftings are as follows: Equation (27), where Equations (28-33).

According to the accepted assumption Equation (3), bending of the plate 2 will be zero (Equation 34). The obtained Equations (13-15) completely determine the deformed state of the lateral plate 2 as well as of the plate 4 of the waveguide at its inertial loading.

CALCULATIONS:

To check the correctness of the obtained expressions, we calculated DM of the waveguide as a whole and its lateral plates of sizes: *L* = 400 m, *H* = 0.0174 mm, *B* = 0.0374 mm and *t* = 0.0012 mm (Figure 1). The waveguide material was AD31 aluminum alloy. The waveguide was fixed with a simple articulated support; for the sake of simplicity, the initial bending moments along the waveguide section edges were taken to be zero: $M_{Z0} = 0$. The acceleration a_Y = 200 m/s^2 was set as external load.

At the first stage, waveguide stresses and deformations were calculated by applying the Euler–Bernoulli beam theory. Then the obtained results were used as boundary conditions at marking out the plate 2 (Figure 2). Further calculating DM of plate 2 was performed using the obtained dependences Equations (7, 11, 13-15).

3.1. Waveguide calculation by the Euler-Bernoulli beam theory

The calculation of waveguide as a whole was performed according to the Euler–Bernoulli beam theory. The normal stresses were calculated from the Navier equation (Timoshenko, 1976) (Equation 35). The shear stresses were calculated from the Zhuravsky shear stress formula (Feodosiev, 1999; Beer *et al.*, 2001; Kecman, 1983) that in the case of waveguide under study is Equation (36, 37). In the beam model, the deformed state of waveguide was determined using the universal Krylov equation (Feodosiev, 1999; Beer *et al.*, 2001) that in our case of loading is Equation (38).

The maximal bending is in the middle of the waveguide beam model (Equation 39). The results of waveguide computation according to the Euler–Bernoulli beam theory are given in summary Table 1.

3.2. Calculation of waveguide lateral plate 2

According to the proposed procedure, let us assign the lateral plate 2 of the waveguide, apply the corresponding actions to its lateral boundaries (Figure 2) and perform the refined calculation of its DM:

1) The shear stress $\tau_2^{'}$ calculated from Equation (17) is applied over the upper and lower plate sides;

2) The pressure of the plates 1 and 3 on the plate 2 at their junctures as normal stresses σ_R calculated as Equation (40) (where γ is the specific weight) is determined as Equation (41).

The calculated weight pressure of the upper (1) and lower (3) plates is σ_R = 9526 Pa. The shear stress $\tau_2^{'}$ at lateral sides of the plates 2 and 4 were obtained by calculating waveguide as a whole from the Zhuravsky formula (known from the Euler–Bernoulli beam theory) and are $\tau_2^{'}$ = 1.6 MPa (Figure 2). The results of computing lateral plate 2 of the waveguide from the obtained dependences Equations (7, 11, 13-15) are given in summary Table 1.

RESULTS AND DISCUSSION:

The main results of calculating stresses and deformations according to the Euler–Bernoulli beam theory and to the proposed procedures as well as their comparison are given in Table 1. The graphics results are presented in Figures 3-6. The obtained results of calculating DM for plate 2 and waveguide as a whole confirmed the reasonableness of the proposed procedures for modeling thin-walled beams as a set of plates. A comparison between the results of calculating stresses and deformations according to the Euler–Bernoulli beam theory and to the proposed procedures showed rather big (up to 14%) discrepancy (see Table 1).

The performed analysis revealed a strong dependence of DM parameters of the plate 2 on the preset shear stresses $\tau_2^{'}$ at the junctures with the plates 1 and 3 (Figure 2). Investigation of $\sigma_{\alpha 2MAX}\left(\tau_2^{'}\right)$ leads to the following expression (Equation (42). By substituting the used waveguide sizes, we get Equation (43).

The $\tau_2^{'}$ value is taken from calculating waveguide as a beam by the Zhuravsky formula. It is evident that application of the same formula to a thin-walled waveguide construction with non-axis-symmetrical cross-section results in a certain inaccuracy in determination of $\tau_2^{'}$ value. According to Equations (22, 23), this leads to distortion of stress $\sigma_{\alpha 2}$ and other DM parameters of the plate 2 (Table 1). So it is necessary to justify a range of applicability of the Zhuravsky formula and reveal the ways to refine the obtained values of shear stresses $\tau_2^{'}$ for thin-walled beams. This is the next stage of the present work after which it will be possible to start estimating DM of the waveguide as a whole.

CONCLUSIONS:

In this work was considered a way for modeling thin-walled beams with a rectangular cross-section. An analysis of deflected mode for extensive thin-walled constructions with non-axis-symmetrical cross-section is very complicated and ambiguous. A new approach was proposed, in which any thin-walled spatial construction is a system of separate elements (plates, shells) joined together. A waveguide presented as a set of plates and subjected to inertial loading served as an example. This approach made it possible to get more correct and exact solution which is of importance for such critical component as a waveguide, as well as to extend realization of the problem of calculating lengthy thin-walled constructions with a non-axis-symmetrical cross-section.

In this paper, only the first part of the solution was presented, because the complete solution for the problem of waveguide DM determination, using the theory was proposed, is very complicated. At the first stage of calculations, we studied deflected mode of a lateral plate 2 whose behavior appreciably determined the state of the waveguide as a whole. The obtained results of calculating DM for plate 2 and waveguide as a whole confirmed the reasonableness of the proposed procedures for modeling thin-walled beams as a set of plates.

The final results confirmed the correctness of the applied approach and directed the lines of further investigations.

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$$\left.\begin{aligned}\nabla^4\varphi_i &= E\cdot\left[\left(\frac{\partial^2\omega_i}{\partial\alpha_i\partial\beta_i}\right)^2-\frac{\partial^2\omega_i}{\partial\alpha_i^2}\cdot\frac{\partial^2\omega_i}{\partial\beta_i^2}\right]+E\alpha\cdot\nabla^2\left[T_i(\alpha_i,\beta_i)-T_{0i}(\alpha_i,\beta_i)\right],\\ \nabla^4\omega_i &= \frac{t}{D}\left[\frac{\partial^2\varphi_i}{\partial\beta_i^2}\cdot\frac{\partial^2\omega_i}{\partial\alpha_i^2}-2\frac{\partial^2\varphi_i}{\partial\alpha_i\partial\beta_i}\cdot\frac{\partial^2\omega_i}{\partial\alpha_i\partial\beta_i}+\frac{\partial^2\varphi_i}{\partial\alpha_i^2}\frac{\partial^2\omega_i}{\partial\beta_i^2}-q_{\alpha i}\frac{\partial\omega_i}{\partial\alpha_i}-q_{\beta i}\frac{\partial\omega_i}{\partial\beta_i}+q_{Zi}-\rho t\frac{\partial^2\omega_i}{\partial\tau^2}\right]\end{aligned}\right\} \quad (1)$$

$$\nabla^4=\frac{\partial^4}{\partial\alpha_i^4}+2\frac{\partial^4}{\partial\alpha_i^2\partial\beta_i^2}+\frac{\partial^4}{\partial\beta_i^4}; \quad (2)$$

$$\left.\begin{aligned}&t\cdot\left.\frac{\partial^2\varphi_{i+1}}{\partial\alpha_{i+1}^2}\right|_{\beta_{i+1}=b_i+1}=-D_i\left.\left(\frac{\partial^3\omega_i}{\partial\beta_i^3}+\frac{\partial^3\omega_i}{\partial\beta_i\partial\alpha_i^2}\right)\right|_{\beta_i=-b_i}; \quad \left.\frac{\partial^2\varphi_{i+1}}{\partial\alpha_{i+1}\partial\beta_{i+1}}\right|_{\beta_{i+1}=b_{i+1}}=\left.\frac{\partial^2\varphi_i}{\partial\alpha_i\,\partial\beta_i}\right|_{\beta_i=-b_i};\\ &-D_i\left.\left(\frac{\partial^3\omega_{i+1}}{\partial\beta_{i+1}^3}+\frac{\partial^3\omega_{i+1}}{\partial\beta_{i+1}\partial\alpha_{i+1}^2}\right)\right|_{\beta_{i+1}=b_{i+1}}=t\cdot\left.\frac{\partial^2\varphi_i}{\partial\alpha_i^2}\right|_{\beta_i=-b_i}; \quad \left.\frac{\partial^2\omega_{i+1}}{\partial\alpha_{i+1}^2}+\mu\frac{\partial^2\omega_{i+1}}{\partial\beta_{i+1}^2}\right|_{\beta_{i+1}=-b_{i+1}}=\left.\frac{\partial^2\omega_i}{\partial\alpha_i^2}+\mu\frac{\partial^2\omega_i}{\partial\beta_i^2}\right|_{\beta_i=-b_i}.\end{aligned}\right\}$$

(3)

$$\theta_{\beta_i}=\frac{\partial\omega_i}{\partial\beta_i}=0. \quad (4)$$

$$\left.\begin{aligned}\nabla^4\varphi_i &= 0;\\ \nabla^4\omega_i &= 0.\end{aligned}\right\}; \quad (5)$$

$$\left.\frac{\partial^2\varphi_{i+1}}{\partial\alpha_{i+1}^2}\right|_{\beta_{i+1}=b_{i+1}}=0; \quad \left.\frac{\partial^2\varphi_i}{\partial\alpha_i^2}\right|_{\beta_i=-b_i}=0; \quad \left.\frac{\partial^2\varphi_{i+1}}{\partial\alpha_{i+1}\partial\beta_{i+1}}\right|_{\beta_{i+1}=b_{i+1}}=\left.\frac{\partial^2\varphi_i}{\partial\alpha_i\,\partial\beta_i}\right|_{\beta_i=-b_i};$$

$$\left.\frac{\partial^2\omega_{i+1}}{\partial\alpha_{i+1}^2}\right|_{\beta_{i+1}=-b_{i+1}}=\left.\frac{\partial^2\omega_i}{\partial\alpha_i^2}\right|_{\beta_i=-b_i} \quad (6)$$

$$\varphi_2(\alpha_2,\beta_2)=\frac{d_5}{6}\alpha_2{}^2\cdot\beta_2{}^3-\frac{d_5}{30}\beta_2{}^5+\frac{d_3}{6}\beta_2{}^3+\frac{b_3}{2}\alpha_2{}^2\cdot\beta_2+\frac{a_2}{2}\alpha_2{}^2. \quad (7)$$

$$\sigma_{\alpha 2}=\frac{\partial^2\varphi_2(\alpha_2,\beta_2)}{\partial\beta_2^2}=d_5\alpha_2^2\beta_2-\frac{2d_5\beta_2^3}{3}+(d_3+\mu\gamma)\beta_2; \quad (8)$$

$$\sigma_{\beta 2}=\frac{\partial^2\varphi_2(\alpha_2,\beta_2)}{\partial\alpha_2^2}=\frac{d_5\beta_2^3}{3}+(b_3+\gamma)\beta_2+a_2; \quad (9)$$

$$\tau_{\alpha 2\beta 2}=-\frac{\partial^2\varphi_2(\alpha_2,\beta_2)}{\partial\alpha_2\partial\beta_2}=-d_5\alpha_2\beta_2^2-b_3\alpha_2. \quad (10)$$

$$\tau_{\alpha 2\beta 2}\left(\beta_2=\pm\frac{h}{2}\right)=\tau_2^{'}. \quad (11)$$

$$\left.\sigma_{\beta 2}\left(\beta_2=+\frac{h}{2}\right)=-\sigma_R; \quad \sigma_{\beta 2}\left(\beta_2=-\frac{h}{2}\right)=+\sigma_R\right\} \quad (12)$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \beta_2 \cdot \sigma_{\alpha 2}(\alpha_{2,}\beta_2) d\beta_2 = M_{Z0} \quad \text{at} \quad \alpha_2 = \pm \frac{L}{2}. \tag{13}$$

$$a_2 = 0; \tag{14}$$

$$b_3 = -\sigma_R \frac{3}{h} - \gamma \frac{3}{2} - \frac{\tau_2^{'}}{L}; \tag{15}$$

$$d_3 = M_{Z0} \frac{12}{h^3} - \mu\gamma - d_5 \frac{5L^2 - 2h^2}{20}; \tag{16}$$

$$d_5 = \frac{6}{h^2}\left(\sigma_R \frac{2}{h} + \gamma + \frac{\tau_2^{'}}{L} \right). \tag{17}$$

$$\varphi_2(\alpha_2, \beta_2) = \alpha_2 \cdot p(\alpha_2, \beta_2) + \beta_2 \cdot q(\alpha_2, \beta_2) + r(\alpha_2, \beta_2), \tag{18}$$

$$\left. \begin{aligned} p(\alpha_2, \beta_2) &= -\frac{d_5}{12}\alpha_2 {\beta_2}^3 + \frac{d_5}{12} \cdot \alpha_2^3 \beta_2 + \frac{b_3 + d_3}{4}\alpha_2 \beta_2; \\ q(\alpha_2, \beta_2) &= -\frac{d_5}{48}{\beta_2}^4 + \frac{d_5}{8} \cdot \alpha_2^2 \beta_2^2 + \frac{b_3 + d_3}{8}\beta_2^2 - \frac{d_5}{48} \cdot \alpha_2^4 - \frac{b_3 + d_3}{8}\alpha_2^2; \\ r(\alpha_2, \beta_2) &= \frac{d_5}{8}{\alpha_2}^2 {\beta_2}^3 - \frac{d_5}{80}{\beta_2}^5 + \frac{3b_3 - d_3}{8}{\alpha_2}^2 \beta_2 + \frac{d_3 - 3b_3}{24}{\beta_2}^3 - \frac{d_5}{16}\alpha_2^4 \beta_2. \end{aligned} \right\} \tag{19}$$

$$u_2(\alpha, \beta) = \Phi_1 - \frac{1}{4(1-\mu)} \frac{\partial}{\partial \alpha_2}(\alpha_2 \Phi_1 + \beta_2 \Phi_2 + \Phi_0); \tag{20}$$

$$v_2(\alpha, \beta) = \Phi_2 - \frac{1}{4(1-\mu)} \frac{\partial}{\partial \beta_2}(\alpha_2 \Phi_1 + \beta_2 \Phi_2 + \Phi_0); \tag{21}$$

$$\left. \begin{aligned} \Phi_0 &= \frac{4(1-\mu)}{2G} r(\alpha_2, \beta_2) = \frac{2(1-\mu)}{G}\left(\frac{d_5}{8}\alpha_2^2 {\beta_2}^3 - \frac{d_5}{80}{\beta_2}^5 + \frac{3b_3 - d_3}{8}\alpha_2^2 \beta_2 + \frac{d_3 - 3b_3}{24}{\beta_2}^3 - \frac{d_5}{16}\alpha_2^4 \beta_2 \right); \\ \Phi_1 &= \frac{4(1-\mu)}{2G} p(\alpha_2, \beta_2) = \frac{2(1-\mu)}{G}\left(-\frac{d_5}{12}\alpha_2 {\beta_2}^3 + \frac{d_5}{12} \cdot \alpha_2^3 \beta_2 + \frac{d_3 + b_3}{4}\alpha_2 \beta_2 \right); \\ \Phi_2 &= \frac{4(1-\mu)}{2G} q(\alpha_2, \beta_2) = \frac{2(1-\mu)}{G}\left(-\frac{d_5}{48}{\beta_2}^4 + \frac{d_5}{8} \cdot \alpha_2^2 \beta_2^2 + \frac{b_3 + d_3}{8}\beta_2^2 - \frac{d_5}{48} \cdot \alpha_2^4 - \frac{b_3 + d_3}{8}\alpha_2^2 \right). \end{aligned} \right\} \tag{22}$$

$$u_2(\alpha_2, \beta_2) = \frac{1}{6G}\left(-A_1 \alpha_2 {\beta_2}^3 + B_1 \alpha_2^3 \beta_2 + C_1 \alpha_2 \beta_2 \right). \tag{23}$$

$$A_1 = d_5(2 - \mu); \tag{24}$$

$$B_1 = d_5(1-\mu);\qquad (25)$$

$$C_1 = 3(d_3 - \mu d_3 - \mu b_3).\qquad (26)$$

$$v_2(\alpha_2,\beta_2) = A_2 \cdot \beta_2^2 + \frac{1}{24G}\left(B_2\beta_2^{\,4} - C_2 \cdot \alpha_2^2\beta_2^2 + D_2\beta_2^2 + E_2 \cdot \alpha_2^4 + F_2 \cdot \alpha_2^2\right),\qquad (27)$$

$$A_2 = \frac{0.5-\mu}{1-\mu} \cdot \frac{\gamma}{2G};\qquad (28)$$

$$B_2 = d_5(1+\mu);\qquad (29)$$

$$C_2 = 6\mu d_5;\qquad (30)$$

$$D_2 = 2(3b_3 - 3\mu b_3 + d_3 - 3\mu d_3);\qquad (31)$$

$$E_2 = d_5(\mu - 1);\qquad (32)$$

$$F_2 = 6(\mu b_3 - 2b_3 - d_3 + d_3\mu).\qquad (33)$$

$$\omega_2(\alpha_2,\beta_2) = 0.\qquad (34)$$

$$\sigma_{X\_MAX}\left(x = \frac{L}{2}, y = \pm\frac{H}{2}\right) = \frac{H}{2J_Z}\left[M_{Z0} + \frac{qL^2}{8}\right].\qquad (35)$$

$$\tau_2^{'}\left(x = 0, y = \pm\frac{h}{2}\right) = \frac{Q_y}{4J_Z} \cdot B(H-t);\qquad (36)$$

$$\tau_{\max} = \tau_3(x=0, y=0) = \frac{Q_y}{4J_Z} \cdot \left[B(H-t) + \frac{h^2}{2}\right].\qquad (37)$$

$$u_y(x) = -\frac{q}{24EJ_Z}\left(-x^4 + 2Lx^3 - L^3x\right) + \frac{M_{Z0}}{2EJ_Z}\left(x^2 - L \cdot x\right).\qquad (38)$$

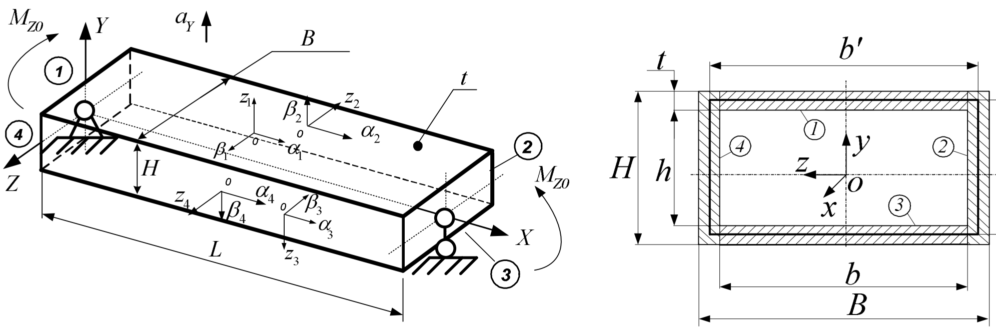
$$u_{y\_MAX}\left(x = \frac{L}{2}\right) = -\frac{L^2}{8EJ_Z} \cdot \left(\frac{5qL^2}{48} + M_{Z0}\right).\qquad (39)$$

$$\sigma_R = \frac{\gamma \cdot L \cdot B}{2(L+B-2t)},\qquad (40)$$

$$\gamma = ma_Y.\qquad (41)$$

$$\sigma_{\alpha 2MAX} = M_{Z0}\frac{6}{H^2} - \sigma_R\left(\frac{2}{5} + \frac{3}{2}\frac{L^2}{H^2}\right) - \gamma\left(\frac{H}{5} + \frac{3}{4}\frac{L^2}{H}\right) - \tau_2^{'}\left(\frac{2}{5}\frac{H}{L} + \frac{3}{2}\frac{L}{H}\right). \quad (42)$$

$$\sigma_{\alpha 2MAX} \sim 35 \cdot \tau_2^{'}. \quad (43)$$



a) design model of waveguide b) waveguide cross-section

Figure 1. *Design model of waveguide straight section.*

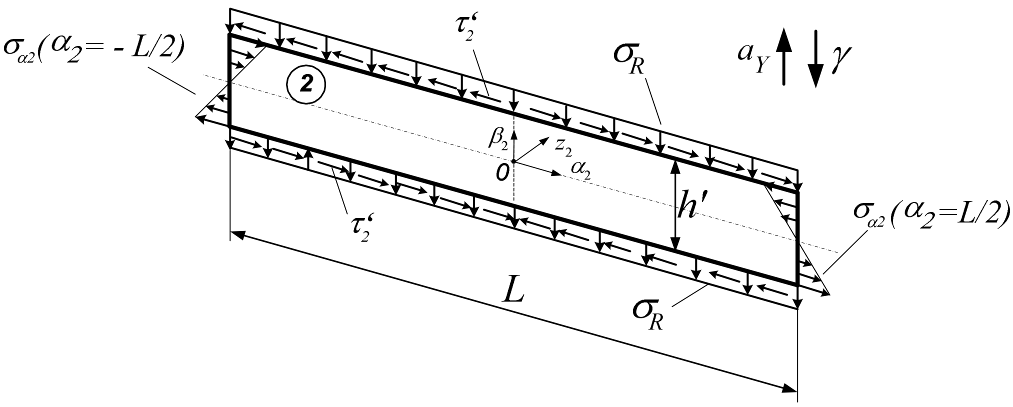


Figure 2. *Design model for a lateral plate of the waveguide.*

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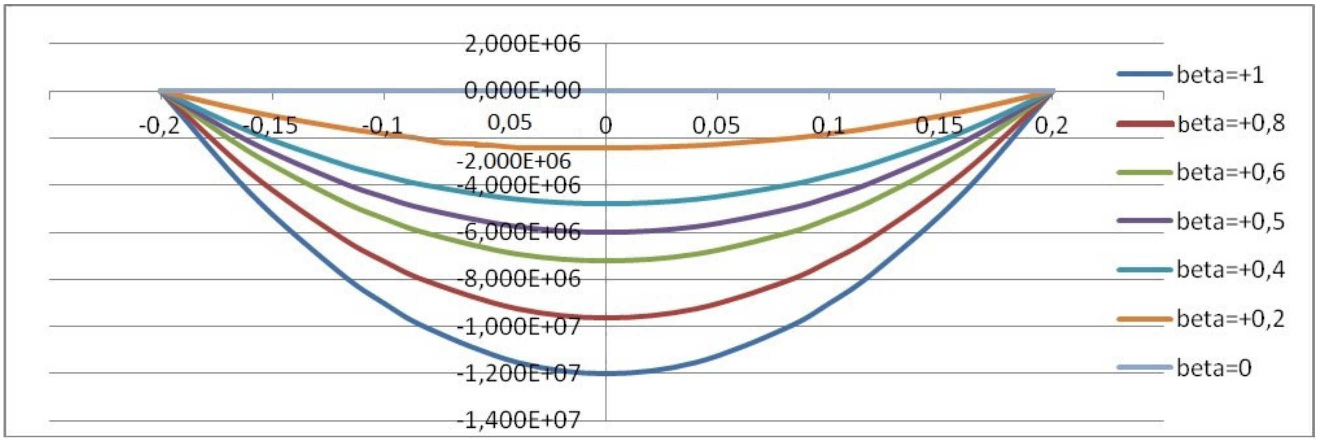

Figure 3. *Diagram of longitudinal stresses* $\sigma_{\alpha 2}(\alpha_2, \beta_2)$ *changing: gradually along plate 2 length and stepwise (in fractions) along plate 2 height (from zero to one half-height).*

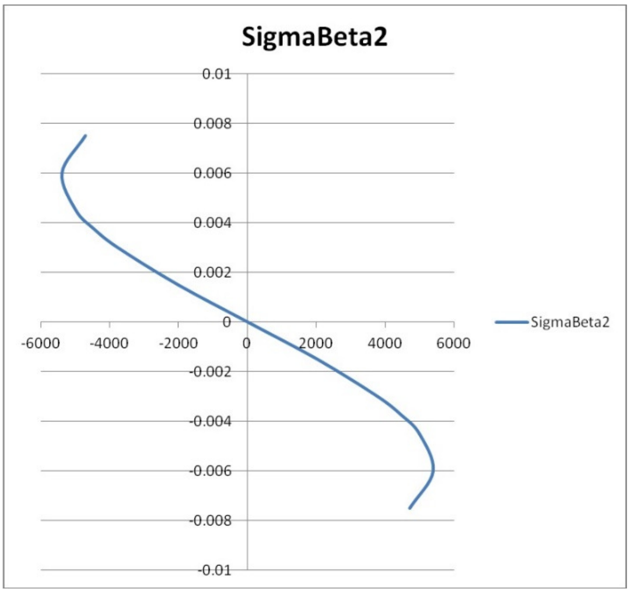

Figure 4*. Diagram of transversal stresses* $\sigma_{\beta 2}(\alpha_2, \beta_2)$ *changing gradually along plate 2 height (there is no changing along plate 2 length).*

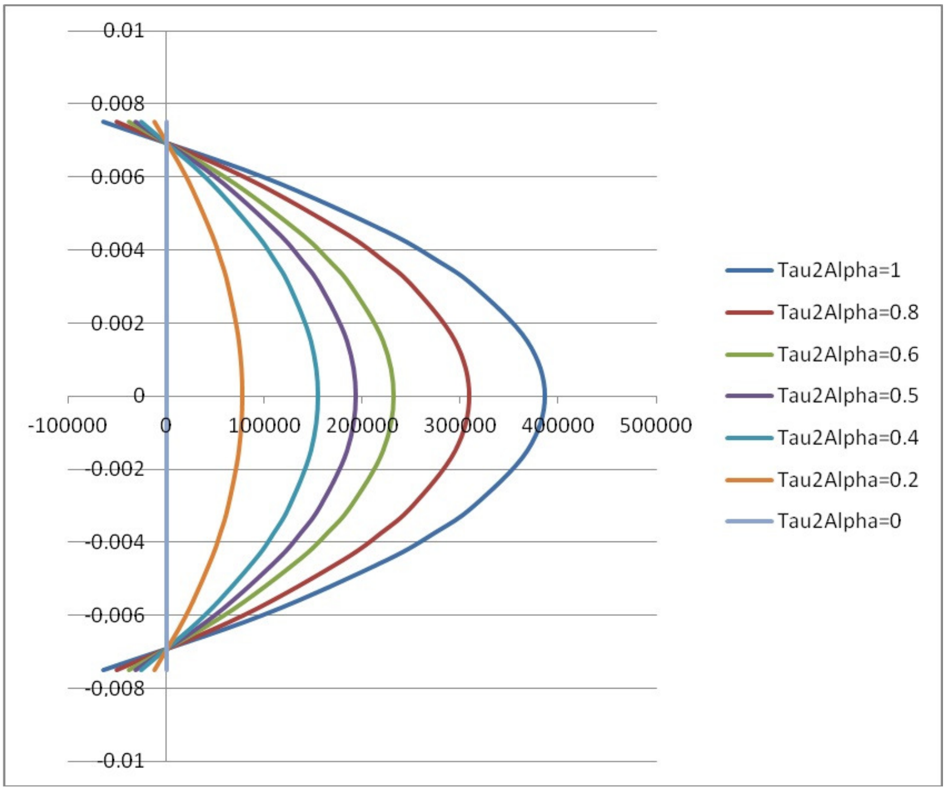

Figure 5*. Diagram of shear stresses $\tau_{\alpha 2 \beta 2}(\alpha_2, \beta_2)$ changing gradually along plate 2 height and stepwise (in fractions) along plate 2 length (from its middle zero to the end one).*

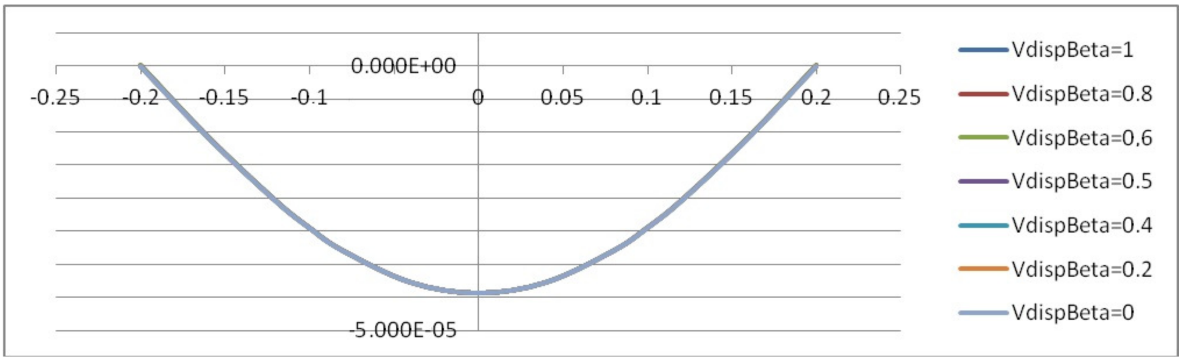

Figure 6*. Diagram of transversal shifting $v_2(\alpha_2, \beta_2)$ gradually along plate 2 length and stepwise (in fractions) along plate 2 height (from zero to one half-height).*

Table 1. *Results of waveguide DM calculation*.

Parameter	$\sigma_{X\_MAX}\left(x=\frac{L}{2}, y=\frac{h}{2}\right)$, Pa	$\tau_{Y\_MAX}(x=0, y=0)$, Pa	$\tau_2'\left(x=0, y=\frac{h'}{2}\right)$, Pa	$u_{y\_MAX}\left(x=\frac{L}{2}\right)$, m
Values from the Euler–Bernoulli beam theory	1 716 629.3	380 614.9	321 009.7	–4.975E-05
Values from the proposed procedure	1 864 621.556	386 745.3		–4.271E-05
Departures	7.9%	1.8%	-	14%