PROBLEMA TRANSITÓRIO PARA A DIFRACÇÃO DA ONDA DE PRESSÃO ACÚSTICA CILÍNDRICA EM UMA MEMBRANA ELÍPTICA FINA, CONSIDERANDO A EFICIÊNCIA DA DISSIPAÇÃO

TRANSIENT PROBLEM FOR DIFFRACTION OF ACOUSTIC CYLINDRICAL PRESSURE WAVE ON THIN ELLIPTIC SHELL CONSIDERING DISSIPATION EFFECT

НЕСТАЦИОНАРНАЯ ЗАДАЧА ДИФРАКЦИИ ЦИЛИНДРИЧЕСКОЙ АКУСТИЧЕСКОЙ ВОЛНЫ ДАВЛЕНИЯ НА ТОНКОЙ ЭЛЛИПТИЧЕСКОЙ ОБОЛОЧКЕ С УЧЕТОМ ЭФФЕКТА ДИССИПАЦИИ

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RESUMO

O artigo considera um problema transitório plano para a dinâmica de uma membrana elíptica elástica fina imersa em um líquido e carregada com ondas de pressão acústicas cilíndricas. Esta situação pode ser observada em vários sistemas de refrigeração de aeronaves e veículos. A solução deste problema é possível com a ajuda do sistema de equações para o problema relacionado. Além disso, os problemas de hidroelasticidade são reduzidos à equação da dinâmica da membrana, onde o efeito do amortecimento do fluido (isto é, dissipação) é considerado pela introdução de uma convolução integral no domínio do tempo. O núcleo do operador é uma função de transição de superfície do problema de difração de uma onda acústica cilíndrica de pressão sobre uma superfície convexa. Uma solução aproximada deste problema pode ser obtida no contexto de uma hipótese de camada fina, considerando a força de amortecimento do líquido. As equações integro-diferenciais correspondentes da dinâmica da membrana são resolvidas numericamente usando a discretização de diferenças de operadores diferenciais, e o operador integral é representado por uma fórmula de integração de trapézio.

Palavras-chave: *onda de pressão, coordenadas curvilíneas, solução numérica, sistema cilíndrico.*

ABSTRACT

We consider a plane transient problem for dynamics of the thin elastic elliptic shell, immersed in liquid and loaded by cylindrical acoustic pressure waves. Such a situation could be observed in various cooling systems of aviation and transport vehicles. In order to solve this problem an equation system for the coupled problem is formulated. Besides, the problems of hydroelasticity are reduced to the equation of shell dynamics, where the liquid damping effect (i.e. the dissipation) is considered by the introduction of the integral convolution in the time domain. The operator core is a surface transient function of the problem of diffraction of a cylindrical acoustic pressure wave on a convex surface. The approximate solution of this problem could be obtained on the background of the thin layer hypothesis considering liquid damping force. The appropriate integrodifferential equations of shell dynamics are solved numerically using the finite difference discretization of the differential operators while the integral operator is represented by the trapezoid integration formula.

Keywords: *pressure wave, curvilinear coordinates, numerical solution, cylindrical system*

АННОТАЦИЯ

В статье рассмотрена плоская переходная задача для динамики тонкой упругой эллиптической оболочки, погруженной в жидкость и нагруженной цилиндрическими акустическими волнами давления. Такая ситуация может наблюдаться в различных системах охлаждения авиационных и транспортных средств. Решение этой задачи возможно с помощью системы уравнений для связанной задачи. Кроме того, проблемы гидроупругости сводятся к уравнению динамики оболочки, где эффект демпфирования жидкости (т.е. диссипация) рассматривается путем введения интегральной свертки во временной области. Ядро оператора является поверхностной переходной функцией задачи дифракции цилиндрической волны акустического давления на выпуклой поверхности. Приближенное решение этой задачи можно было бы получить на фоне гипотезы тонкого слоя, рассматривающей жидкую демпфирующую силу. Соответствующие интегро-дифференциальные уравнения динамики оболочек решаются численно с использованием разностной дискретизации дифференциальных операторов, а интегральный оператор представлен формулой интегрирования трапеции.

Ключевые слова: *волна давления, криволинейные координаты, численное решение, цилиндрическая система.*

INTRODUCTION

One of the most urgent problems of modern mechanics consists of the study of the transient interaction of shock waves spreading in continuous media with various deforming obstacles. The research works in this field is of considerable interest both from the point of view of the development of mathematical methods for solving initial boundary value problems of mechanics, and for a number of technical applications, in particular, the calculation of thin-walled structural elements loaded with shock waves in a liquid (Dmitriev *et al.*, 2017; Dmitriev *et al.*, 2015; Kuznetsova *et al.*, 2018; Danilin *et al.*, 2016; Kakhramanov *et al.*, 2017).

Here we study the dynamic behavior of thin-walled elastic isotropic shells immersed in a liquid and subjected to acoustic shock waves, while the main attention is paid to the construction of approximate models of the interaction of a deforming shell with a wave diffracting on it. For this purpose, damping in a liquid is taken into account (Kupatadze and Kizilöz, 2016). The transition functions, i. e. fundamental solutions of the non-stationary initial boundary value problem of diffraction of an acoustic medium on a smooth convex surface is the main mathematical apparatus developing in the presented work. The application of transition functions provides a transition from the solution of the associated non-stationary problem of the joint motion of an acoustic medium and a deforming obstacle to the solution of the problem only for an obstacle, the mathematical model of which takes into account interaction with the external environment in the form of integral relations. The kernels of the integral terms of the equations of obstacle motion are formed on the grounds of the transition functions of the diffraction problem (Gorshkov *et al.*, 2003; Lomakin *et al.*, 2017; Krupenin *et al.*, 2014; Rabinskiy *et al.*, 2016). Thus, the dimensionality of the problem is reduced, which makes it possible to significantly simplify the numerical solution on the basis of a finite-element or finite-difference approach, and in some important special cases to develop analytical solutions and estimate the errors introduced by the hypotheses that are accepted. The solution to this problem is shown on the example of determining the kinematic parameters in a thin elliptical shell during the diffraction of a cylindrical acoustic pressure wave.

MATERIALS AND METHODS

The diffraction of weak shock waves in a liquid on an elastic elliptical shell is considered. This paper investigates transient processes in a liquid for which heat exchange can be neglected. In this case, a model of a barotropic ideal fluid can be used, the system of equations of motion of which is nonlinear in the absence of mass forces and the constancy of the entropy. The lenarization of this system of equations leads to a model of the acoustic medium, which reduces to the corresponding wave equation. In this case, dissipation in a liquid is taken into account (Rabinskiy and Zhavoronok, 2018; Lurie *et al.*, 2015; Lurie *et al.*, 2011). An isotropic material is used as the material of an elliptical shell and the problem can be considered in a linear formulation. To solve this problem, the integral Laplace transforms with respect to time is used with subsequent conversion using tables and its properties. As a result, the initial boundary value problem is integrated using the finite difference method in an explicit scheme.

RESULTS AND DISCUSSION:

3.1. Features of shock wave diffraction

The diffraction of the weak shock wave in the liquid is studied based on approximate models. The solution of the problem is based on transient function apparatus being an elementary solution of transient initial-boundary value for diffraction of the acoustic area on the plane-convex surface.

We consider the problem on diffraction of the nonstationary cylindrical wave on thin elastic basket surface, immersed in acoustic area. The transient function built on the basis of thin layer hypothesis (Medvedskiy and Rabinskiy, 2007; Gorshkov *et al.*, 2003a; Gorshkov *et al.*, 2003b; Zhavoronok *et al.*, 2010) is used to compute the hydraulic pressure loading the shell. The integration of Timoshenko motion equations, constructed with the help of Maple 9.0, is performed by the finite difference method of with Matlab 6.5.

3.2. Scenario

The mathematical task scenario appears as follows. Acoustic medium (Equations 1, 2) (Medvedskiy and Rabinskiy, 2007). Elastically isotropic thin shell (Equations 3 – 6) (Zhavoronok, 2018). Here, φ – speed potential in the acoustic area, p – pressure in reflected and emitted waves, $\mathbf{v}$ – acoustic area velocity vector, u_i – generalized displacements of middle surface of a shell, $\mathbf{L}_{ij}$ – known differential operators, indicating by shell geometry, δ_{ij} – Kronecker's delta. Relations (Equation 5) are indicated with the help of operators $\mathbf{N}^{(m)}(u_i)$ boundary condition, depending on shell form and its space attachment, β-parameter, indicating liquid dissipation.

Further, the problem is solved in a dimensionless form. Besides, all linear sizes are related to the length of major semi-axis of elliptic shall a, speed is related to sound speed within the acoustic area c_0, values having pressure dimension are related to $\rho_0 c_0^2$ complex, time τ is related to tc_0/a.

In reference time $\tau = 0$ shell and area are in a non-perturbed state that leads to homogenous initial conditions Equation 2 and Equation 4.

Suppose that originally, cylindrical wavefront with front pressure p_0 touches the shell in any point A (Figure 1). Besides, wave source is situated in K with $(-c, -b)$ coordinates.

Based on conditions of coupled motion of shell and adjacent particles of acoustic area impermeability conditions are written as follows (Equation 6), where φ_* – velocity potential of the wave falling to the shell, $\partial/\partial n$ – derivative on external normal to the shell, w – shell inflection.

Pressures p_1 and p_2 in reflected and emitted waves can be calculated with the help of transient function $G(x^i, \tau)$, constructed within the thin-layer hypothesis (an asterisk denotes fold operation considering time τ) (Equations 7 – 9). Besides, influence function $G(x^i, \tau)$ satisfies the following initial-boundary value (Equations 10 – 12), where $\delta(\tau)$ is the Dirac delta function.

3.3. Hydrodynamic pressure on shell's surface

We introduce the curvilinear chart (ξ^1, ξ^3), related to curve Γ. Let $\mathbf{r}_0(\xi^1)$ be the radius vector of the point on Γ, and $\mathbf{n}_0(\xi^1)$ – normal unit (Figure 1). Then, the coordinates (ξ^1, ξ^3) of the curvilinear frame are represented as follows (differentiation is further denoted by lower index) Equation 13. Metric tensor components will be written hence as follows Equation 14, where $k = k(\xi^1)$ is the curvature of the curve Γ.

Let us assume that the main contribution to hydrodynamic load gives medium motion on normal to surface Medvedskiy and Rabinskiy, 2007; Gorshkov *et al.*, 2003a; Gorshkov *et al.*, 2003b; Zhavoronok *et al.*, 2010) as a first approximation; therefore we could neglect the tangential motion of a liquid along Γ the surface. Thus, the coordinate derivatives ξ^1 in (Equation 1) can be set identically equal to zero and the Laplace operator can be computed on the cylinder surface $\xi^3 = 0$. The last one corresponds to the Laplace operator Δ_ξ in (Equation 10). Consequently, the initial boundary value problem (Equations 10-12) can be rewritten as follows Equations 13 – 17. The transient function $G_0(\xi^1, \tau)$ on obstacle surface Γ is found by operational method (Gorshkov *et al.*, 2003a) (Equation 18, 19, 20). For this case the formulae for the pressure in reflected and emitted waves can be represented as follows (Equations 21, 22, 23). Here the relations (Equation 7) – (Equation 8) are taken into account.

3.4. Diffraction of the cylindrical wave on the elastic elliptic cylinder

We consider the problem for diffraction of the cylindrical wave, excited by continuously long linear source, which intensity is homogeneous along the axis.

The perturbation source is located at the point K with coordinates ($-c, -b$) in the frame Ox^1x^2, the constant d denotes wavefront position at the initial time point (Figure 1). Let's consider any local Oy^1y^2 coordinates, such as radial coordinates of the cylindrical system as follows (Equation 24). The acoustic wave in initial time moment $\tau = 0$ touches the cylinder surface with a guiding line Γ at A a point. Hydrodynamic pressure on the shell surface is defined by the sum $p = p_* + p_1 + p_2$, where p_* – the pressure of ongoing wave, p_1 – reflected wave pressure, p_2 – radiation pressure.

Pressure and velocity of the cylindrical wave are found on the background of the known ratio (Equations 25 – 27). Complete elliptic integrals of the first and second kind (Equation 28). In this expression $r = r(\xi^1, \xi^3)$ – a cylindrical wave radius will be determined as follows (Equation 29). For the constant d we have the following Equation 30, 31, where ξ_0^1 is the parameter corresponding to the tangency point A. The pressure p_1 in reflected wave is defined by (Equation 25) with correspondent transient function $G_p(\xi^1, \tau)$ (Equation 25) and velocity $v_*(\xi^1, \tau)$ (Equation 26). Consequently, using the curvilinear coordinates we obtain the following formula for the pressure p_1 (Equation 32). The emitted wave pressure p_2 is introduced as follows (Equation 33). The elliptical shell guiding line Γ is parameterized as it is shown below (Equation 34). The dynamic equation for shell can be written as follows (Equation 35, 36). suitable for the numerical solution of the problem discrete analog (here $\mathbf{L}$ – linear operator of the problem, $\mathbf{p}$ – vector-function of right parts) (Zhavoronok, 2018; Pobedrya and Gergievskiy, 1999; Bakhvalov *et al.*, 2003; Kondratov *et al.*, 2018; Babaev and Kubenko, 1977; Gorshkov and Tarlakovsky, 1990).

In general, construction of governing Equations 35, 36 in curvilinear coordinates, attached to the surface of arbitrary shape, is rather difficult. At the same time, use of Computer Algebra Systems, supporting basic operations of tensor analysis, allows automating transition process from the general notation to its operator notation in specific coordinates. Maple 9.0 CAS is used below with extension pack Tensor.

The obtained results are demonstrated in Figures 2 – 3 for steel elliptic shell (density $\rho = 7200\,\text{кг/м}^3$, modulus of elasticity $E = 2 \cdot 10^6$ MPa, Poisson ratio $\nu = 0,3$, shell thickness $h = 0,01$ m, parity relation $b/a = 0.5$), immersed in water (density $\rho_0 = 1000\,\text{кг/м}^3$, acoustic speed $c_0 = 330\,\text{м/с}$, β). Pressure intensity on ingoing wavefront in reference time $p_0 = 10^4$ Па. Wave source is placed in K point with coordinates $b = -1\,м,\ c = -2\,м$

In Figures 2 – 3 the dependencies of the total pressure and deflection from coordinate at dimensionless time points $\tau = 0.4$, 0.6, 0.625, 1.0. The dashed line shows the same curves considering liquid damping.

CONCLUSIONS:

As a result of the performed calculations, it may be concluded that the summary pressure on the surface of the elliptical shell with accounting for damping in a liquid decreases. A similar effect is observed in determining the deflection of the shell. A critical analysis of the existing methods for constructing exact and approximate solutions of the diffraction problem of a weak shock wave in an acoustic medium on a deformable convex obstacle, as well as the studies presented in this paper showed the effectiveness of using the apparatus of transition functions, which are defined here explicitly and given by Equations 18, 19, 20.

Here we consider the nonstationary problem of the dynamics of an acoustic medium, which is formulated in a curvilinear orthogonal coordinate system, normally associated with an obstacle, an elastic elliptical shell. In this case, the dynamics of the elastic shell and the acoustic medium are set in a single coordinate system, which greatly simplifies the task.

Based on the solution of the problem of diffraction of an acoustic wave on a curvilinear convex obstacle, hypotheses are substantiated, which allow analytically constructing a transition function of this problem that can be used to solve more complex problems, including studies of elastic shells of arbitrary shape and various types of incident waves.

On the basis of the constructed transition functions, the integrodifferential equations of motion of an elastic elliptical shell that is compliant to shear under the action of weak shock waves of various shapes in an acoustic medium are obtained. The interaction with the surrounding continuous medium is modeled by the integral terms of the equations.

For the numerical solution of the integro-differential equations of shell motion, difference schemes are constructed and their convergence is investigated.

From the solution of the problem, it can be seen that taking damping into a fluid slightly affects the pressure diagram (Figure 2) and the shell deflection (Figure 3); fundamental importance.

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$$\frac{\partial^2\varphi}{\partial\tau^2}+2\beta\frac{\partial\varphi}{\partial\tau}=\Delta\varphi,\ \ p=-\frac{\partial\varphi}{\partial\tau},\ \ \mathbf{v}=\mathrm{grad}\varphi \tag{1}$$

$$\varphi\big|_{\tau=0}=\frac{\partial\varphi}{\partial\tau}\bigg|_{\tau=0}=0 \tag{2}$$

$$\frac{\partial^2 u_i}{\partial\tau^2}=\mathbf{L}_{ij}(u_j)+(p_*+p)\delta_{i3}\quad (i,j=1,2,3) \tag{3}$$

$$u_i\big|_{\tau=0}=\frac{\partial u_i}{\partial\tau}\bigg|_{\tau=0}=0 \tag{4}$$

$$\mathbf{N}^{(m)}(u_i)\big|_{\xi^1=\xi^1_k}=0\qquad (k=1,2) \tag{5}$$

$$\frac{\partial w}{\partial\tau}=\frac{\partial\varphi_*}{\partial n}\bigg|_\Gamma+\frac{\partial\varphi}{\partial n}\bigg|_\Gamma \tag{6}$$

$$p_1(\xi^1,\tau)=\frac{\partial\varphi_*(\xi^1,0,\tau)}{\partial n}*G_p(\xi^1,\tau) \tag{7}$$

$$p_2(\xi^1,\tau)=\frac{\partial w}{\partial t}(\xi^1,\tau)*G_p(\xi^1,\tau) \tag{8}$$

$$p=p_1+p_2,\qquad G_p(\xi^1,\tau)=-\frac{\partial G(x^i,\tau)}{\partial\tau}\bigg|_\Gamma \tag{9}$$

$$\frac{\partial^2 G}{\partial\tau^2}+2\beta\frac{\partial G}{\partial\tau}=c_0^2\Delta_\xi G \tag{10}$$

$$G\big|_{\tau=0}=\frac{\partial G}{\partial\tau}\bigg|_{\tau=0}=0 \tag{11}$$

$$\frac{\partial G}{\partial n}\bigg|_\Gamma=\delta(\tau),\ \ G(r,\tau)=O(1)\ \text{при}\ r\to\infty \tag{12}$$

$$\mathbf{r}(\xi^1,\xi^1)=\mathbf{r}_0(\xi^1)-\xi^3\mathbf{n}_0(\xi^1) \tag{13}$$

$$g_{11}=H_1^2=\boldsymbol{\tau}^2\left[1+2\xi^3k+\left(\xi^3k\right)^2\right],\quad g_{12}=0,\ \ g_{22}=H_2^2=1, \tag{14}$$

$$\frac{\partial^2 G}{\partial \tau^2} + 2\beta \frac{\partial G}{\partial \tau} = \frac{c_0^2}{H_1} \left[\frac{\partial}{\partial \xi^1} \left(H_1 \frac{\partial G}{\partial \xi^1} \right) \right] \Bigg|_{\xi^3=0} \tag{15}$$

$$G\big|_{\tau=0} = \frac{\partial G}{\partial \tau}\bigg|_{\tau=0} = 0 \tag{16}$$

$$\frac{\partial G}{\partial \xi^3}\bigg|_{\xi^3=0} = \delta(\tau), \;\; G(r,t) = O(1) \text{ при } r \to \infty \tag{17}$$

$$G_0(\xi^1,\tau) = k\left(\xi^1\right)\Phi_1\left(\tau\right) - \Phi_2\left(\tau\right) - k\left(\xi^1\right)^2 \int_0^\tau \Phi_1\left(\tau - t\right)\Phi_2\left(\tau\right)dt \tag{18}$$

$$\Phi_1(\tau) = \frac{1 - e^{-2\beta\tau}}{2\beta} \;\;, \tag{19}$$

$$\Phi_2(\tau) = e^{-2\beta\tau} J_0\left(k\left(\xi^1\right)^2 - \beta^2\right) \tag{20}$$

$$p_1(\xi^1,\tau) = -\int_0^\tau \frac{\partial \varphi_*(\xi^1, 0, \tau - t)}{\partial \xi^2} G_p(\xi^1, t)dt \tag{21}$$

$$p_2(\xi^1,\tau) = -\int_0^\tau \frac{\partial u_1(\xi^1, \tau - t)}{\partial t} G_p(\xi^1, t)dt \tag{22}$$

$$G_p(\xi^1,\tau) = \frac{\partial G_0(\xi^1,\tau)}{\partial \tau} \tag{23}$$

$$r^2 = \left(y^1\right)^2 + \left(y^2\right)^2 \tag{24}$$

$$p_*(r,\tau) = -\dot{\varphi}_*(r,\tau) = \frac{2\sqrt{2}}{\pi} \frac{K(\mathrm{m})}{\sqrt{\tau + r}} H(\tau - r)\,; \tag{25}$$

$$v_*(r,\tau) = \frac{\partial \varphi_*(r,\tau)}{\partial r} = $$
$$= -\sqrt{\frac{2}{\pi}} \frac{1}{r\sqrt{\pi\left(\tau + r\right)}} \left[rK(m) - \left(\tau + r\right)E(m) \right] H(\tau - r), \tag{26}$$

$$K(m)=\int_0^{\pi/2}\frac{dt}{\sqrt{1-m\sin^2 t}},\quad E(m)=\int_0^{\pi/2}\sqrt{1-m\sin^2 t}\,dt. \tag{27}$$

$$m=\sqrt{\frac{\tau-r}{\tau+r}} \tag{28}$$

$$r(\xi^1,\xi^3)=\sqrt{\left(x^1(\xi^1,\xi^3)-c\right)^2+\left(x^2(\xi^1,\xi^3)-b\right)^2}\,. \tag{29}$$

$$\sqrt{\left(x_0^1(\xi_0^1)-c\right)^2+\left(x_0^2(\xi_0^1)-b\right)^2}+d=0; \tag{30}$$

$$x_0^1(\xi_0^1)'(x_0^1(\xi_0^1)-c)+x_0^2(\xi_0^1)'(x_0^2(\xi_0^1)-b)=0\,, \tag{31}$$

$$p_1(\xi^1,\tau)=-\int_0^{\tau} v_*(\xi^1,\tau)G_p(\xi^1,\tau-t)dt \tag{32}$$

$$p_2(\xi^1,\tau)=-\int_0^{\tau} w(\xi^1,\tau)G_p(\xi^1,\tau-t)dt \tag{33}$$

$$\Gamma:\ x_0^1(\xi^1)=-a\cos\xi^1,\ \ x_0^2(\xi^1)=a\sin\xi^1,\ \ \xi^1\in\left[-\pi,\pi\right) \tag{34}$$

$$\frac{\partial^2\mathbf{u}}{\partial\tau^2}=\mathbf{Lu}+\mathbf{p} \tag{35}$$

$$\mathbf{L}=\mathbf{C}\frac{d}{d\xi^2}+\mathbf{B}\frac{d}{d\xi}+\mathbf{A} \tag{36}$$

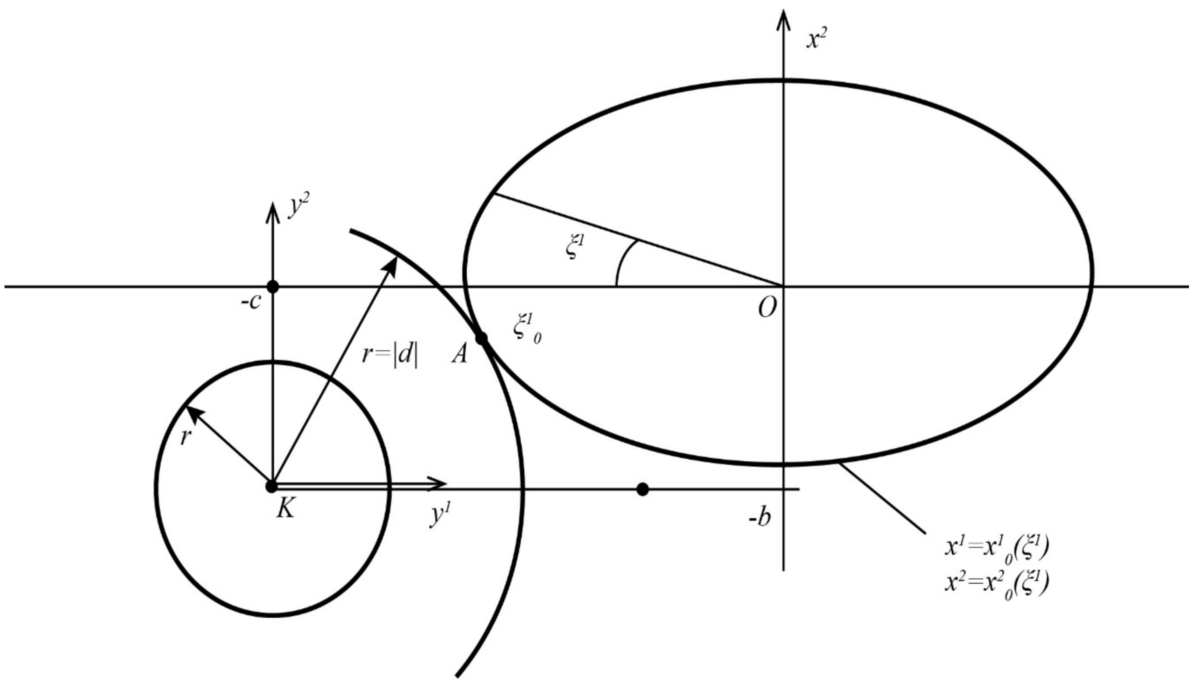

Figure 1. *Diffraction of a non-stationary cylindrical wave on a thin elastic elliptical shell*

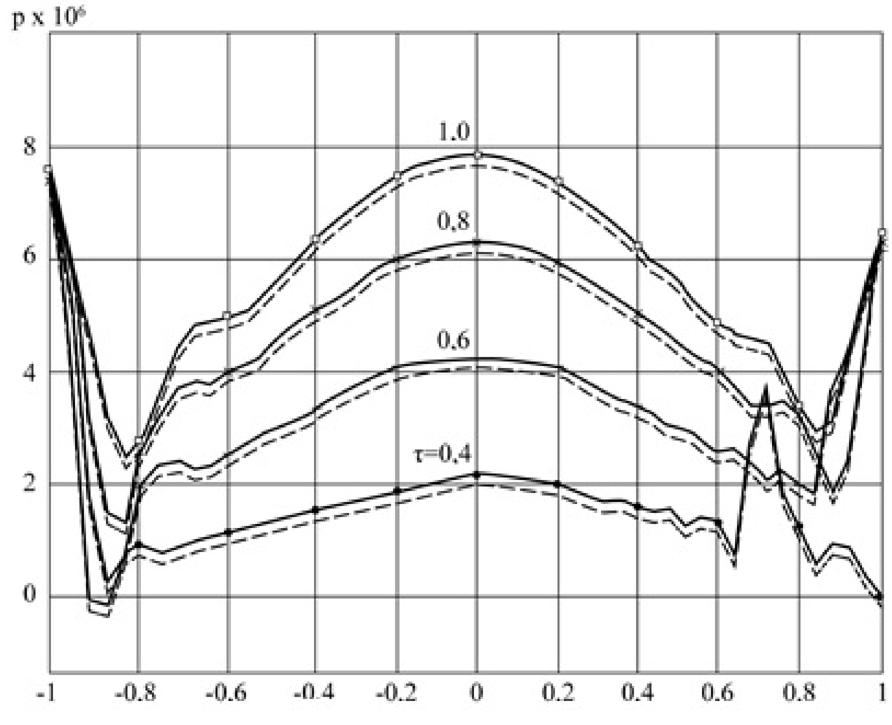

Figure 2. *Summary pressure on the surface of the elliptical shell at different points of time*

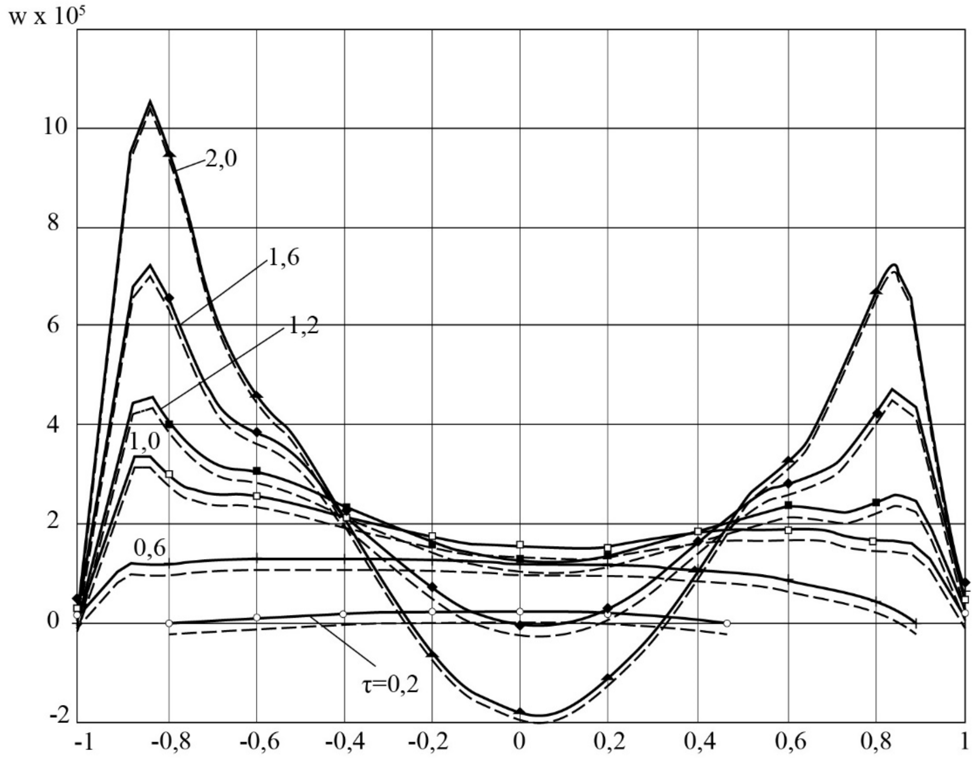

Figure 3. *Deflection of the elliptical shell at different points of time*