DINÂMICA DA ESTRUTURA DE LAJES FINAS COM ALTO NÍVEL DE ELASTICIDADE E OS SUPORTES ELÁSTICOS DISCRETOS EM UMA SUPERFÍCIE RÍGIDA COM CARGAS EM MOVIMENTO

DYNAMICS OF THIN-WALLED STRUCTURE WITH HIGH ELONGATION RATIO AND DISCRETE ELASTIC SUPPORTS ON THE RIGID SURFACE UNDER MOVING LOADS

ДИНАМИЧЕСКОЕ ПОВЕДЕНИЕ УДЛИНЕННОЙ ТОНКОСТЕННОЙ КОНСТРУКЦИИ ДИСКРЕТНО УПРУГО ПРИКРЕПЛЕННОЙ К ЖЕСТКОЙ ПОВЕРХНОСТИ ПОД ДЕЙСТВИЕМ ПОДВИЖНОЙ НАГРУЗКИ

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Received 15 June 2018; received in revised form 26 November 2018; accepted 05 December 2018

RESUMO

Em posições dinâmicas e quase-estáticas, o problema da deformação não-estacionária de uma estrutura de lajes finas estendida, que é presa a uma superfície rígida, é resolvido no processo de aplicação de uma carga adicional. O compartimento da estrutura estendida é modelado por meio de uma viga. As propriedades do revestimento de proteção de calor estão incluídas na equação de oscilações de flexão da viga através de forças inerciais. O trabalho descreve os danos locais usando funções genéricas. A carga móvel é simulada por uma força linear infinitamente distribuída uniformemente que se move ao longo da viga com uma velocidade constante. Como resultado, as forças inerciais têm uma estrutura mais complexa do que na formulação clássica. O problema é reduzido à equação diferencial de oscilações de flexão da viga em derivadas parciais. A velocidade da carga é incluída na equação como um parâmetro. Para a solução, utiliza-se o método de Bubnov, segundo o qual a deflexão de uma viga é representada como uma série de determinadas funções coordenadas com coeficientes desconhecidos, que são considerados como coordenadas generalizadas. Utilizando os métodos de experimento computacional, foi investigada a possibilidade de reduzir os valores máximos de aceleração em determinados pontos dos sistemas aeroespaciais sob a influência de diferentes cargas estáticas e dinâmicas. A velocidade crítica de movimento da carga foi calculada.

Palavras-chave: *estrutura de lajes finas, fixadores, modelo dinâmico simplificado, experimento computacional, velocidade crítica de movimento.*

ABSTRACT

In dynamic and quasistatic positions the problem on nonstationary deformation of extended thin-wall construction discreetly elastically attached to the rigid surface is approximately solved under live load. The properties of the heat-insulating shield are included in the equation of flexural vibrations of the beam through inertia forces. Local damage is described using generalized functions. The movable load is simulated by an infinite uniformly distributed normal force per unit length moving along the beam at a constant speed. Consequently, the inertial forces have a more complex structure than in the classical presentation. The problem is limited to the differential equation of flexural vibrations of the beam in partial derivatives. The speed of the load is included in the equation as a parameter. For the solution, the Bubnov method is used, in accordance with which the beam deflection is represented as a series of given coordinate functions with unknown

DOI: 10.52571/PTQ.SI1.2018.481_PeriodicoES.pdf

coefficients, which are considered as generalized coordinates. The possibility of decreasing framed structures maximum acceleration values in set points of aerospace systems under the action of different static and dynamic loads is investigated with the methods of computing experiment. Critical speed of load movement is calculated.

Keywords*: thin-wall construction, attachment joints, simplified dynamic model, computing experiment, critical movement speed*.

АННОТАЦИЯ

В динамических и квазистатических положениях проблема нестационарной деформации расширенной тонкостенной конструкции, которая прикреплена к жесткой поверхности, решается в процессе применения дополнительной нагрузки. Отсек удлиненной конструкции моделируется балкой. Свойства теплозащитного покрытия входят в уравнение изгибных колебаний балки через силы инерции. В работе локальные повреждения описаны с помощью обобщенных функций. Подвижная нагрузка имитируется бесконечной равномерно распределенной погонной силой, которая движется вдоль балки с постоянной скоростью. Вследствие этого инерционные силы имеют более сложную структуру, чем в классической постановке. Задача сводится к дифференциальному уравнению изгибных колебаний балки в частных производных. Скорость движения нагрузки входит в уравнение в качестве параметра. Для решения используется метод Бубнова, в соответствии с которым прогиб балки представлен в виде ряда заданных координатных функций с неизвестными коэффициентами, которые рассматриваются в качестве обобщённых координат. С применением методов вычислительного эксперимента исследована возможность уменьшения максимальных значений ускорения в заданных точках аэрокосмических систем под действием разных статических и динамических нагрузок. Рассчитывается критическая скорость движения груза.

Ключевые слова: *тонкостенная конструкция, крепежные соединения, упрощенная динамическая модель, вычислительный эксперимент, критическая скорость движения*.

INTRODUCTION

The work is devoted to the investigation of the possibility of reducing the maximum values of accelerations at given points of frame structures of aerospace launching systems under the action of static and dynamic loads of various types (Wright, 2017; Hashemi, 2016; Chen *et al.*, 2013; Groh and Pirrera, 2018). With increasing speeds of aerospace systems, there is a need to improve the calculation methods and to create more and more accurate methods for solving the problems of dynamic interaction of thin-walled structures under the action of movable loads (Danilin *et al.*, 2015; Lotfy, 2016; Baimakhan *et al.*, 2016; Fan *et al.*, 2017; Wei *et al.*, 2017; Lomakin *et al.*, 2018).

An attempt is made here to develop efficient numerical methods and algorithms for solving the problems of the dynamics of long-span beams with variable parameters simulating the elastic base under the action of movable railway loads or other non-stationary impacts.

At the first stage, the problem of dynamic deformation of a two-layer composite elongated thin-walled structure under the action of movable inertial load, normal to its axis, is approximately solved (Freitas and Loeffler, 2016; Yuan *et al.*, 2016). The compartment of the extended construction is modeled with a beam. In this case, inertial forces have a more complex structure than in the case when the beam deflection depends only on its longitudinal coordinate (Liu *et al.*, 2019).

With regard to the above mentioned, the problem is limited to the differential equation of flexural vibrations of the beam in quartic partial derivatives. For the solution, the Bubnov method is used, according to which the deflection of the beam is represented as a series of given coordinate functions with unknown coefficients, which are considered as generalized coordinates (Tian *et al.*, 2015). As an example, in a quasistatic formulation in the binomial approximation, the lower critical speeds of the load motion for various forms of shell stability loss.

MATERIALS AND METHODS

An approximate solution to the dynamic

behavior of extended thin-wall construction discreetly elastically attached to the rigid surface under live load. Such type problems appear in case of pressure wave influence on side missile boosters discreetly attached to the main carrying block. During exploitation, they are adjacent to both standard and local bends in the area of attachment joints (Manannikov *et al.*, 2017; Gasparrini *et al.*, 2018; Elder *et al.*, 2017). As a result, for a description of structural dynamic behavior, we use a simplified accounting model based on the possibility of deformed state division on standard and local one. The first of it is associated with the bend of the extended shell as a beam with non-deforming transversal contour and the second one is associated with its local flexibility in the area of block joints. In accordance with it, we consider beam shell model, suspended near terminal sections $x = \pm\xi$ on equivalent tension-compression spring (Figure 1), which rigidity c_r (*r* = 1,2) is indicated by local shell flexibility in the area of attachment joints. Besides, its inertia can be neglected because of moving localness (Loeffler *et al.*, 2013; Ruben *et al.*, 2017). The moving load influencing on beam we can model by continuous stripe of normal evenly distributed intensity load *p* moving at a constant speed *V*, conditionally specified in the figure as force per unit length.

RESULTS AND DISCUSSION:

First of all, the authors solve the problem considering dynamic formulation. We assume that beam bends depend not only on its longitudinal coordinate x but time *t*. Besides, the deflection curve $w(x,t)$ is simultaneously loaded movement trajectory. As for specified time *t* live load element will go the distance $x = Vt$, then the speed projection of this element dw/dt for normal to the beam axis and its vertical acceleration d^2w/dt^2 will be full derivatives Equations 1,2. The second formula item for acceleration contains mixed derivative corresponding to Coriolis acceleration and in the process of practical problems, it's usually neglected (Konoplev and Yakushev, 2003; Panovko and Gubanova, 1979). The rigidity of equivalent spring *c* can be found as value reversible to its local flexibility *f (c=1/f).* The last one can be approximately indicated as cylindrical shell deflection under influence of concentrated radial single forces Equation 3 (Nerubaylo, 1983). Where *R, h* and *E* – radius, shell thickness and module of elasticity of its material accordingly.

Concentrated standard forces appearing within block attachment joints should be specified within the Winkler hypothesis. In light of all of the above for a description of the dynamic deformed state of simplified beam model of shell we use its lateral oscillation equation as follows Equation 4, where *E* and *J* – beam material elasticity modulus and its cross-section inertia accordingly, m_0 – bulk weight, $\delta(x-\xi_r)$ – Dirac delta function with the reference position $x = \xi_r (r = 1,2)$ of block attachment joints (equivalent spring attachment coordinates), g – acceleration of gravity.

Equation 4 as a parameter contains load movement speed *V*. For its approximation solve we use Bubnov method in accordance with which we demonstrate beam deflection as the following factorization Equation 5. Where $w_1(t) = \delta(t)$ and $w_2(t) = \theta(t)$ – displacement and beam turning angle as a rigid body at the beginning of coordinates (Figure 1). Approximating functions $\varphi_1 = 1$ and $\varphi_2 = x$ correspond to it. The other unknown time functions $w_i(t)$ indicate flexural beam deformations, $\varphi_i(x)$ - set coordinate functions. Substituting factorization (Equation 5) in Equation 4 and using Bubnov method we lead the problem to the system of ordinary differential equations of the second order towards unknown functions w_i in factorization (Equation 5). In matrix form is has the following view Equation 6. Where *M* and $K_{(V)}$ – a square matrix of mass and beam rigidity, and *W* and *P* – vectors of unknown functions $w_i(t)$ and gravity loads accordingly Equation 7. Elements of this matrix and vectors are as follows in Equation 8, 9, 10. In Equation 8 function primes φ_i denote its derivatives according to *x* axis. Because of accounting of beam shift as rigid body setting by functions $\varphi_1 = 1$ and $\varphi_2 = x$ even in the process of selection of the other approximating functions orthogonal matrix of rigidity *K* and mass *M* will not be diagonal one. So, a solution of the Equation 6 in high approximations may be numerically indicated only. As the elements of this stiffness matrix include load movement speed, then for some of its values called critical ones, beam deflections begin incrementally increase, that may be considered as the loss of structural stability (Gorshkov et al., 2003; Zhavoronok et al., 2010; Medvedskiy and Rabinskiy, 2007).

When solving the problem in a simpler quasistatic position, we assume that beam deflection does not depend on time and it can be calculated on longitudinal coordinate *x*. Then, the equation of lateral beam vibrations (Equation 4) acquires a simpler form (Equation 11).

All elements of Equation 11 are standard. To solve it we also use Bubnov method, demonstrating beam deflection *w* as factorization Equation 5 but under w_i we understand not only time functions but unknown coefficients. After Bubnov method use Equation 11 goes to simultaneous linear algebraic equations towards constants w_i. In matrix form, it has the following view (Equation 12).

Rigid matrix elements $K_{(V)}$ and vectors P are calculated according to Equation8. The peculiarity of these equations is its dependence on movement load speed. The critical value of this speed is calculated based on the condition of zero equality of system determinant (Equations 12, 13).

Examples. We consider the framework structure, representing an extended shell attached by frontal sections to the rigid stationary surface with equal equivalent springs. For this system in quasistatic formation in binomial approximation, we indicate minimum critical load speed on different forms of shell stability loss. In case of symmetric or anti-symmetric form towards the origin of coordinates (Figure 1) stability loss forms beam deflection can be accordingly represented as Equations 14, 15.

In these approximations, the first items are associated with beam movement as a rigid body and the second ones are associated with its rigid deformations. Beam movements as a rigid body and elastic couplings influence only on its movement and critical speed only depend on bending deformations. Squares of minimum dimensionless critical speeds $V_{KP}^{2*} = V_{KP}^{2} p / Egl^{2}$ found following (Equation 12) in case of asymmetrical and anti-symmetric form of stability loss are accordingly equal Equations 16, 17.

So, stability loss on anti-symmetric form, as could be expected, occurs in case of the lower speed of movement.

CONCLUSIONS:

Thin models tend to flex under axial load. Loss of stability is defined as the sudden deformation that occurs when the stored membrane (axial) energy is converted to bending energy without changing the applied external loads. Mathematically, when a loss of stability occurs, the stiffness becomes degenerate. The linearized buckling method used here solves the eigenvalue problem to estimate critical buckling factors and the corresponding forms of buckling mode. The model can be bent in different forms under loads of various levels. The form that the model assumes during buckling is called the form of buckling mode, and the load is called "critical" or "critical longitudinal load."

Loss of stability of real shells in many cases occurs at a lower load due to the significant influence of various factors, especially initial shape irregularities.

For complex structures, the exact solution is difficult, so they resort to various approximate methods. For many of them, the energy sustainability criterion is used, in which the nature of the change in the potential energy of the system is considered when it is little deviated from the equilibrium position (for stable equilibrium P = min). When considering non-conservative systems, for example, a rod compressed by a force whose slope changes in the process of buckling (tracking force), a dynamic criterion is used, consisting in determining small oscillations of the loaded system. Of importance is the study of so-called. supercritical behavior of elastic systems. It requires solving nonlinear boundary value problems. For a rod, supercritical deformation is possible only with its very large flexibility. On the contrary, for thin plates, significant deflections in the supercritical stage are quite possible - provided that the edges of the plate are supported by rigid rods (stringers). For shells, supercritical deformation is usually associated with snapping and loss of the bearing capacity of the structure.

As a result of the calculations performed, the lower critical speeds of the load motion for various forms of shell stability loss were determined for the binomial approximation. It is concluded that the shell stability loss occurs in an antisymmetric form and is realized at a lower speed of the load.

ACKNOWLEDGMENTS:

This work is executed with funding from RFFI (project No 16-01-00641). This work was supported by the Grant of the President of the Russian Federation MK-1204.2017.8.

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$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}, \tag{1}$$

$$\frac{d^2 w}{dt^2} = \frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial x \partial t} + V^2 \frac{\partial^2 w}{\partial x^2}. \tag{2}$$

$$f = 3(\frac{R}{h})^2 \frac{1}{ER}\sqrt{\frac{R}{h}} \tag{3}$$

$$EJ\frac{\partial^4 w}{\partial x^4} + \frac{p}{g}V^2\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial t^2}(m_0 + \frac{p}{g}) + \sum_{r=1}^{2} wc_r \delta(x - \xi_r) = m_0 g + p \tag{4}$$

$$w(x,t) = \sum_{i=1}^{N} w_i(t)\varphi_i(x), \tag{5}$$

$$M\frac{d^2 W}{dt^2} + K_{(V)}W = P \tag{6}$$

$$M = [m_{ij}], \quad K_{(V)} = [k_{ij(V)}], \quad W = \{w_j\}, \quad P = \{p_j\}. \tag{7}$$

$$m_{ij} = (m_0 + \frac{p}{g})\int_{-l}^{l} \varphi_i \varphi_j dx, \tag{8}$$

$$p_j = (mg + p)\int_{-l}^{l} \varphi_j dx, \tag{9}$$

$$k_{ij(V)} = EJ\int_{-l}^{l} \varphi_i^{IV} \varphi_j dx + \frac{p}{g}V^2 \int_{-l}^{l} \varphi_i'' \varphi_j dx, + \sum_{r=1}^{2} c_r \int_{-l}^{l} \varphi_i \varphi_j \delta(x - \xi_r) dx \tag{10}$$

$$EJ\frac{d^4 w}{dx^4} + \frac{p}{g}V^2 \frac{d^2 w}{dx^2} + \sum_{r=1}^{2} wc_r \delta(x - \xi_r) = m_0 g + p \tag{11}$$

$$K_{(V)}W = P \tag{12}$$

$$\det\left[K_{(V)}\right] = 0. \tag{13}$$

$$w = \delta + w_1 \cos\frac{\pi x}{2l}, \tag{14}$$

$$w = \theta x + w_1 \sin\frac{\pi x}{l}. \tag{15}$$

$$V_{KP}^{2*} = \frac{J\pi^2}{l^4}\ , \tag{16}$$

$$V_{KP}^{2*} = \frac{J\pi^2}{4l^4}. \tag{17}$$

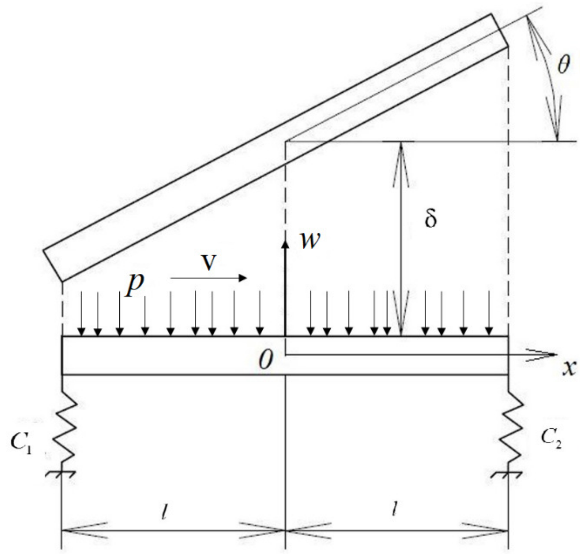

Figure 1. *The beam model of the shell, suspended near the frontal sections*