PESQUISA ANALÍTICA DE TRANSFERÊNCIA TÉRMICA EM ESPAÇO ANISOTRÓPICO COM COMPONENTES DE TEMPERATURA DE CONDUTIVIDADE TÉRMICA DEPENDENDO DA TEMPERATURA

ANALYTICAL STUDY ON HEAT TRANSFER IN ANISOTROPIC SPACE WITH THERMAL CONDUCTIVITY TENSOR COMPONENTS DEPENDING ON TEMPERATURE

АНАЛИТИЧЕСКОЕ ИССЛЕДОВАНИЕ ТЕПЛОВОЙ ПЕРЕДАЧИ В АНИЗОТРОПИЧЕСКОМ ПРОСТРАНСТВЕ С КОМПОНЕНТАМИ ТЕРМИЧЕСКОЙ ПРОВОДИМОСТИ В ЗАВИСИМОСТИ ОТ ТЕМПЕРАТУРЫ

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RESUMO

No contexto deste trabalho, baseado na solução analítica construída do problema de transferência térmica em corpos anisotrópicos, componentes do tensor de condutividade térmica, dependendo da temperatura, a transferência térmica é investigada sob a influência de uma fonte pontual de calor dentro do corpo ou em uma superfície plana. Descobriu-se que a transferência térmica tem um comportamento ondulatório com a presença de uma frente de temperatura, onde as derivadas temporais e as variáveis espaciais são destruídas, mas são contínuas em relação à temperatura e ao fluxo de calor. Além disso, as frentes de onda de calor são elipsoides no espaço e elipsoides planos, dependendo do tempo. É possível calcular as coordenadas da velocidade da onda de calor, enquanto as velocidades dos pontos frontais são diferentes em diferentes direções.

Palavras-chave: *condutividade térmica não-linear, corpos anisotrópicos, componentes do tensor de condutividade térmica, solução analítica, ondas de calor.*

ABSTRACT

Within this work, based on the built analytical solution of heat transfer problem in anisotropic bodies, the thermal conductivity tensor components depending on temperature, the heat transfer is investigated under influence of a point heat source, inside a body or on a plane body surface. It turned out that heat transfer has a wave-like behavior with the presence of temperature front, where derivatives with respect to time and spatial variables are broken but they are continuous with respect to temperatures and heat flux. Besides, the heat wavefronts are ellipsoids in space and plane ellipsoids depending on time. Coordinates of heat wave velocity can be calculated, at that, velocities of front points are different in different directions.

Keywords: *nonlinear thermal conductivity, anisotropic bodies, thermal conductivity tensor components, analytical solution, heat waves.*

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АННОТАЦИЯ

В рамках этой работы, основанной на построенном аналитическом решении задачи теплообмена в анизотропных телах, компонентах тензора теплопроводности в зависимости от температуры, теплопередача исследуется под влиянием точечного источника тепла внутри тела или на поверхности плоского тела. Оказалось, что передача тепла имеет волнообразное поведение с наличием температурного фронта, где производные по времени и пространственные переменные разрушаются, но они являются непрерывными по отношению к температурам и тепловому потоку. Кроме того, тепловые волновые фронты являются эллипсоидами в космических и плоских эллипсоидах, зависящих от времени. Можно вычислить координаты скорости тепловой волны, при этом скорости фронтальных точек различна в разных направлениях.

Ключевые слова: *нелинейная теплопроводность, анизотропные тела, компоненты тензора теплопроводности, аналитическое решение, тепловые волны.*

INTRODUCTION

Heat transfer in bodies, thermal and physical characteristics of which depend on temperature (nonlinear medium), has wave-like behavior even in cases when thermal conductivity is based on Fourier phenomenology (relaxation phenomena are absent) (Samarskii et al., 1995; Zarubin and Kuvyrkin, 2002; Paz *et al.*, 2016). This is fully related to anisotropic bodies with the difference that the fronts of thermal waves depending on time are ellipsoids in space and ellipses on a plane and have different velocities of thermal wave points fronts.

The peculiarity of heat transfer in anisotropic bodies with general anisotropy (absence of non-zero tensor components) is the presence of mixed derivatives in heat transfer equations (Vidyarthi and Singh, 2016; Nenkaew and Tangthieng, 2016). This fact is significantly troubled the obtaining of analytical solutions for anisotropic heat transfer problems even in non-linear cases (Lasebikan *et al.*, 2015; Singh and Sinha, 2017). The analytical solution of anisotropic thermal conductivity problems with general anisotropy can be obtained by integral methods in open or semi-open areas (Formalev, 2012; Formalev *et al.*, 2018).

In a paper (Zarubin *et al.*, 2018) the analytical solution of anisotropic thermal conductivity problem is found based on the variation method, particularly transversal anisotropy.

Thermal conductivity nonlinear problem solution peculiarities in one-dimensional regions are fully investigated in monograph (Kozdoba *et al.*, 1975; Benzeggouta *et al.*, 2018; Lurie *et al.*, 2017; Lomakin *et al.*, 2018), besides, wave activities of nonlinear thermal conductivity are omitted there, but clear nonlinear classification is given. Nonlinearity, connected with the dependence from the temperature of thermal and physical characteristics is called non-linearity of the first kind.

In reference (Polyanin and Zajcev, 2002) many analytical solutions of quasilinear equations of diffusion in the one-dimensional region are given based on the implementation of automodel variables and general method of variables separation, however, without any initial and boundary conditions.

In this paper, based on a formed chain of the automodel variables the analytical solution of thermal conductivity in anisotropic mediums of which thermal conductivity tensor components depend on temperature is found and investigated, however, these components are homogenous polynomials depending on temperature.

Investigation of the analytical solution showed that the thermal conductivity has a wave-like behavior with clear-cut fronts depending on time, at that, different temperature points on fronts move with different velocities (anisotropy sequence). In the linear case, a temperature profile is monotonically progressively reduced in accordance with the infinite velocity of heat flux (Juhl, 2016; Hong *et al.*, 2012).

In details thermal conductivity in anisotropic bodies is considered in reference (Formalev, 2014; Formalev, 2015; Formalev and Kolesnik, 2018; Formalev and Kolesnik, 2007; Formalev and Kolesnik, 2016a; Formalev and Kolesnik, 2016b; Formalev *et al.*, 2006; Formalev and Kolesnik, 2017a; Formalev and Kolesnik, 2017b).

METHODOLOGY

The problem of non-stationary distribution of temperatures $T(x,y,z,t)$ in 3-D anisotropic space V from an instantaneous point source with energy E_0, applied at the origin of coordinates $x=0,\ y=0,\ z=0$ at an initial time $t=0$, i.e. the following Cauchy problem (Equations 1, 2). where $\delta(x-0),\ \delta(y-0),\ \delta(z-0)$ – Dirac delta function, E_0 – pulse energy, besides, space integral from temperature distribution initiated by this energy is constant value Equation 3 where initial condition can be indicated as follows Equation 4. So, instead of the initial condition (Equation 2) the condition (Equation 4) is considered, so, solution of the problem (Equation 1), (Equation 4) will depend on $E_0 / c\rho$. Tensor components of the thermal conductivity in equation (1) are defined by expressions Equation 5 where the main tensor components of thermal conductivity depend on temperature as follows Equation 6. and $\alpha_{ij},\ i=\{\xi,\eta,\zeta\},\ j=\{x,y,z\}$ – direction cosines of angles between the main axis $O\xi,\ O\eta,\ O\zeta$ of the thermal conductivity tensor and axis $Ox,\ Oy,\ Oz$ of rectangular Cartesian coordinate system; $k_\xi = \text{const},\quad k_\eta = \text{const},\quad k_\zeta = \text{const}.$

It's needed to find the non-stationary distribution of temperatures $T(x,y,z,t)$ under influence of the point source (Equation 4), applied to the point with the following coordinates $x=0,\ y=0,\ z=0$ at the initial time $t=0$, besides, in the other space points in accordance with condition (Equation 2) temperature is equal to zero.

We use the linear transformation of rotation around the origin of the Cartesian coordinate system up to coincidence with the main tensor axis of thermal conductivity, indicating by the following ratios (Equations 7 – 9). Since matrix of the linear transformation (Equation 8) is not singular and orthogonal, then inverse matrix coincides with transpose one, consequently, based on formulas (Equation 8) we can get inverted transformation Equations 10 – 12. Substitution of transformation (Equation 8) to the problem (Equations 1, 3, 4) leads to the problem for the equation, where mixed differential operators are absent Equations 13 – 15. Move to a new system of coordinates (Equation 16) where L – any number (for example, $L=1$). From Equations 14, 15 we can obtain Equations 17 – 19. Here, $a = L / c\rho$.

We'll seek the solution of problem (18), (19) in automodel view Equation 20, where Equation 21. In Equations 22, 23 indexes α and β are defined by substitution Equations 22, 23 to the problem (Equations 18, 19). Then we'll obtain Equations 22, 23. Based on Equations 22, 23 the following two expressions can be written for α and (Equations 24, 25, 26). Considering Equation 26, the problem (Equations 22, 23) transforms into the following stationary form (Equations 27, 28).

Suppose the function $\theta(q_1,q_2,q_3)$ is centrally symmetric, i.e., depends on one coordinate r of the spherical coordinate system (Equations 29 – 34). Then, the problem (Equations 27, 28) transforms to the following Cauchy problem for nonlinear ordinary differential Equations 35, 36. During the transition to a spherical coordinate system (Equations 29-34) the integral (Equation 28) over variable δ is equal to 2π, over variable γ – 2 and Jacobian is equal $r^2\cos\gamma$. Since the function $\theta(q_1,q_2,q_3)$ is centrally symmetric and transforms to the function $\theta(r)$, then, for Equation 35 symmetry condition should satisfy (Equation 37). So, the problem (Equations 35, 36) for nonlinear general differential Equation 35 is represented as Equations 38, 39, 40. The first integral of Equation 38 will be Equation 41. Besides, due to Equation 40 at $r=0$ a constant value of integration C_1 is equal to zero. Consequently (Equation 42, 43). Equation 41 is a non-linear ordinary differential equation with separable variables. Its general solution is as follows at Equation 44. Here, a constant value of integration is defined by expression $\frac{r_0^2}{2a(3\sigma+2)}$, and r_0^2 can be calculated using condition (Equation 43). Considering (44) we'll find (Equation 45), where $r\in[0;r_0]$. Integral of left-hand expression (Equation 45) is calculated in quadratures, wherefore let's transform it to the view ($\frac{r}{r_0}$ ranges from $0/r_0=0$ to $r_0/r_0=1$). In accordance with (Dwight, 1961) we have Equation 47, where $\Gamma(s)$ – gamma-function, $p = 1/\sigma$, $m = 2$. Simplifying the expression (Equation 47) and substituting it to Equation 46, we'll find constant value r_0^2 (Equation 48). Equations 44 and 48 indicates the solution of Cauchy problem (Equations 42 – 44) and through (Equation 20) – solution (Equations 18, 19). Returning to Cartesian coordinates, we'll find the solution of the original problem at Equations 1, 2 (Equation 49). Here r_0^2 is defined by Equation 48. Based on solution (Equation 49) we may see that if expression in square brackets is equal to zero, then, quadric surface (Equation 50) indicates moving front, separating the area with non-zero temperature and the other part of space with zero temperature (Ose and Kunugi, 2014; Ayoola *et al.*, 2018; Hariramakrishnan and Sendilkumar, 2018). Based on Expression 48 we may see that this area is ellipsoid with half-axes (Equation 51) along the main axis $O\xi$, $O\eta$, $O\zeta$ of the thermal conductivity tensor. So, (Equation 48) indicates moving the front of the heat wave, separating area of nonzero solution and area free of thermal perturbation.

From Equation 48 and Equation 49 we can see that there's no solution at $\sigma = -2/3$ and $\sigma = 0$. Besides, from condition $p + 1 = \frac{1}{\sigma} + 1 > 0$ in (Equation 47) it follows that $\sigma \in (-\infty; -1) \cup (-1; -2/3) \cup (0; \infty)$, and also arguments of gamma functions in Equation 48 should not be negative integer numbers or zero, i.e. $\frac{1}{\sigma} \neq -n$ and $\frac{\sigma + 2}{2\sigma} \neq -n$, $n = 0, 1, 2, ...$ Negative values of σ lead to infinity of the thermal conductivity in problem with zero initial temperature. Thus, the only interval $\sigma \in (0; \infty)$. makes sense.

RESULTS AND DISCUSSION:

The investigation of the solution (Equation 49) in 3-D is difficult and uncomfortable to be performed. So, based on the specified method, the two-dimensional problem in the plane is obtained. It has the following form Equation 52. where φ – angle between the axis $O\xi$ and Cartesian axis Ox, but r_0^2 is represented by Equation 53. Based on the investigation of the condition $\frac{1}{\sigma} + 1 > 0$ in Equation 32 and $\sigma \neq -1$, $\sigma \neq 0$ there are intervals for σ, (Equation 54) where the solution (Equation 52) exists. Based on zero equality in (Equation 52) expressions in square brackets the geometrical place of points in the plane can be Equation 55 being a conic curve, which indicates moving front of the wave, separating area of the non-zero solution and another area with an initial distribution of temperature (zero). So, the fronts of traveling heat wave over cold space in plane represent ellipses with half-axis (Equation 56). From the investigation of Equation 52, it's followed that on the front of traveling wave *the temperature is continuous*, *first-order derivatives* with respect to special variables are *discontinuous, heat flux density* due to the presence of multiplier T^{σ} *are continuous and second-order derivatives* of temperature with respect to 3-D derivatives are *discrete*.

In Figures 1–3, temperature distributions along each of coordinate axis at fixed time are shown based on Equations 52, 53 with the following input data: $\sigma = 1$; $\sigma = 2$; $\sigma = 3$; $\varphi = 0$; $k_{\xi} = 5$ W/(m·$\mathrm{K}^{\sigma+1}$); $k_{\eta} = 1$ W/(m·$\mathrm{K}^{\sigma+1}$); $c\rho = 2000$ J/(m^{3}K), E_0 =1000 J. Results are given in the form of axial sections $y = 0$ and $x = 0$. Besides, value $E_0 / c\rho$ in accordance with Equation 38 is included in expression for r_0^2, which indicates boundaries of heat wavefront. Figures confirm wave-like behavior of temperature distribution with the heat wavefront on isotherm $T = 0$, describing by ellipses with half-axis (Equation 56). At $t \to 0 + 0$ solution (Equation 52) tends to delta-function.

It's clear that nonzero solution areas in different time are areas, bounded by ellipses with different velocities of propagation of heat waves in different directions.

For each angle φ from Equation 52, we can find coordinates x of the wavefront at $y = 0$ and coordinates y at $x = 0$, when $T(x, y, t) = 0$ (Equations 57, 58) where heat wavefront velocity can be found (Equations 59, 60).

These results show that the nearer time to initial time moment the higher temperature and when increasing time it significantly drops. So, at $\sigma=1$ in time moment $t=10^{-6}\text{s}$ temperature is about $3000\,\text{K}$, at the origin of coordinates and at $t=10^{-5}\text{s}$ and $\sigma=1$, $T=1000\text{K}$. For nanosecond durations, the temperature can significantly exceed the phase transformation temperature of any materials at the origin of coordinates. In case of increasing σ the temperature significantly drops, because increasing thermal conductivity leads the thermal energy more intensively transfers to periphery from the place of the energy source application.

CONCLUSIONS:

1. The method of getting of analytical solution of heat conduction problems with nonlinear thermal and physical characteristics in anisotropic space from pulse source of thermal energy was explained.

2. It turned out that the heat transfer process has a wave-like behavior. It is defined by thermal conductivity as homogenous polynomial depending on temperature with the definite front of a heat wave, moving non-stationary over cold space. Besides, in accordance with anisotropic behavior of heat transfer, heat wavefronts, moving in different directions with different velocities are 3-D ellipsoids and 2-D ellipsoids.

3. It's found that temperature is continuous along the fronts of heat waves, first and second temperature derivatives with respect to space derivatives are discrete and densities of heat flux are continuous because of multiplier availability T^{σ}, that's unexpected because first-order derivatives are discrete and heat flows are continuous.

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$$c\rho\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda_{xx}(T)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda_{yy}(T)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda_{zz}(T)\frac{\partial T}{\partial z}\right)+2\frac{\partial}{\partial x}\left(\lambda_{xy}(T)\frac{\partial T}{\partial y}\right)+$$
$$+2\frac{\partial}{\partial x}\left(\lambda_{xz}\frac{\partial T}{\partial z}\right)+2\frac{\partial}{\partial y}\left(\lambda_{yz}(T)\frac{\partial T}{\partial z}\right),\ \{x,y,z\}\in\ (-\infty;\infty),\ t>0, \quad (1)$$

$$E(x,y,z,0)=E_0\cdot\delta(x-0)\delta(y-0)\delta(z-0), \quad (2)$$

$$c\rho\iiint_V T(x,y,z,t)\,dxdydz=E_0=\text{const}, \quad (3)$$

$$\iiint_V T(x,y,z,t)\,dxdydz=\frac{E_0}{c\rho}=\text{const}. \quad (4)$$

$$\begin{aligned}
&\lambda_{xx}(T)=\lambda_\xi(T)\alpha_{\xi x}^2+\lambda_\eta(T)\alpha_{\eta x}^2+\lambda_\zeta(T)\alpha_{\zeta x}^2,\\
&\lambda_{xy}(T)=\lambda_{yx}(T)=\lambda_\xi(T)\alpha_{\xi x}\alpha_{\xi y}+\lambda_\eta(T)\alpha_{\eta x}\alpha_{\eta y}+\lambda_\zeta(T)\alpha_{\zeta x}\alpha_{\zeta y},\\
&\lambda_{xz}(T)=\lambda_{zx}(T)=\lambda_\xi(T)\alpha_{\xi x}\alpha_{\xi z}+\lambda_\eta(T)\alpha_{\eta x}\alpha_{\eta z}+\lambda_\zeta(T)\alpha_{\zeta x}\alpha_{\zeta z},\\
&\lambda_{yy}(T)=\lambda_\xi(T)\alpha_{\xi y}^2+\lambda_\eta(T)\alpha_{\eta y}^2+\lambda_\zeta(T)\alpha_{\zeta y}^2,\\
&\lambda_{yz}(T)=\lambda_{zy}(T)=\lambda_\xi(T)\alpha_{\xi y}\alpha_{\xi z}+\lambda_\eta(T)\alpha_{\eta y}\alpha_{\eta z}+\lambda_\zeta(T)\alpha_{\zeta y}\alpha_{\zeta z},\\
&\lambda_{zz}(T)=\lambda_\xi(T)\alpha_{\xi z}^2+\lambda_\eta(T)\alpha_{\eta z}^2+\lambda_\zeta(T)\alpha_{\zeta z}^2,
\end{aligned} \quad (5)$$

$$\lambda_\xi=k_\xi T^\sigma,\ \ \lambda_\eta=k_\eta T^\sigma,\ \ \lambda_\zeta=k_\zeta T^\sigma, \quad (6)$$

$$\xi=\alpha_{\xi x}x+\alpha_{\xi y}y+\alpha_{\xi z}z, \quad (7)$$

$$\eta=\alpha_{\eta x}x+\alpha_{\eta y}y+\alpha_{\eta z}z, \quad (8)$$

$$\zeta=\alpha_{\zeta x}x+\alpha_{\zeta y}y+\alpha_{\zeta z}z. \quad (9)$$

$$x=\alpha_{\xi x}\xi+\alpha_{\eta x}\eta+\alpha_{\zeta x}\zeta, \quad (10)$$

$$y=\alpha_{\xi y}\xi+\alpha_{\eta y}\eta+\alpha_{\zeta y}\zeta, \quad (11)$$

$$z=\alpha_{\xi z}\xi+\alpha_{\eta z}\eta+\alpha_{\zeta z}\zeta. \quad (12)$$

$$c\rho\frac{\partial T}{\partial t}=\frac{\partial}{\partial\xi}\left(k_\xi T^\sigma\frac{\partial T}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(k_\eta T^\sigma\frac{\partial T}{\partial\eta}\right)+\frac{\partial}{\partial\zeta}\left(k_\zeta T^\sigma\frac{\partial T}{\partial\zeta}\right), \quad (13)$$

$$\{\xi,\eta,\zeta\}\in(-\infty;\infty),\ \ t>0; \quad (14)$$

$$\iiint_V T(\xi,\eta,\zeta,t)\,d\xi d\eta d\zeta=\frac{E_0}{c\rho}=\text{const}. \quad (15)$$

$$x_1=\xi\left(L/k_\xi\right)^{1/2},\ \ x_2=\eta\left(L/k_\eta\right)^{1/2},\ \ x_3=\zeta\left(L/k_\zeta\right)^{1/2}, \tag{16}$$

$$\frac{\partial T}{\partial t}=a\frac{\partial}{\partial x_1}\left(T^\sigma\frac{\partial T}{\partial x_1}\right)+a\frac{\partial}{\partial x_2}\left(T^\sigma\frac{\partial T}{\partial x_2}\right)+a\frac{\partial}{\partial x_3}\left(T^\sigma\frac{\partial T}{\partial x_3}\right), \tag{17}$$

$$\left\{x_1,x_2,x_3\right\}\in\ \left(-\infty;\infty\right),\ \ t>0; \tag{18}$$

$$\frac{\sqrt{k_\xi k_\eta k_\zeta}}{L^{3/2}}\iiint\limits_V T\left(x_1,x_2,x_3,t\right)dx_1dx_2dx_3=\frac{E_0}{c\rho}. \tag{19}$$

$$T\left(x_1,x_2,x_3,t\right)=t^\alpha\cdot\theta\left(q_1,q_2,q_3\right), \tag{20}$$

$$t^{\alpha-1}\left[\alpha\cdot\theta\left(q_1,q_2,q_3\right)-\beta\left(q_1\frac{\partial\theta}{\partial q_1}+q_2\frac{\partial\theta}{\partial q_2}+q_3\frac{\partial\theta}{\partial q_3}\right)\right]=$$
$$=t^{\alpha(\sigma+1)-2\beta}a\left[\frac{\partial}{\partial q_1}\left(\theta^\sigma\frac{\partial\theta}{\partial q_1}\right)+\frac{\partial}{\partial q_2}\left(\theta^\sigma\frac{\partial\theta}{\partial q_2}\right)+\frac{\partial}{\partial q_3}\left(\theta^\sigma\frac{\partial\theta}{\partial q_3}\right)\right]; \tag{22}$$

$$t^{\alpha+3\beta}\frac{\sqrt{k_\xi k_\eta k_\zeta}}{L^{3/2}}\iiint\limits_V\theta\left(q_1,q_2,q_3\right)dq_1dq_2dq_3=\frac{E_0}{c\rho}. \tag{23}$$

$$\alpha-1=\alpha\left(\sigma+1\right)-2\beta, \tag{24}$$

$$\alpha+3\beta=0, \tag{25}$$

$$\alpha=\frac{-3}{3\sigma+2},\ \ \beta=\frac{1}{3\sigma+2}. \tag{26}$$

$$a\frac{\partial}{\partial q_1}\left(\theta^\sigma\frac{\partial\theta}{\partial q_1}\right)+a\frac{\partial}{\partial q_2}\left(\theta^\sigma\frac{\partial\theta}{\partial q_2}\right)+a\frac{\partial}{\partial q_3}\left(\theta^\sigma\frac{\partial\theta}{\partial q_3}\right)+$$
$$+\frac{1}{3\sigma+2}\left(q_1\frac{\partial\theta}{\partial q_1}+q_2\frac{\partial\theta}{\partial q_2}+q_3\frac{\partial\theta}{\partial q_3}\right)+\frac{3}{3\sigma+2}\ \ \theta=0, \tag{27}$$

$$\frac{\sqrt{k_\xi k_\eta k_\zeta}}{L^{3/2}}\iiint\limits_V\theta\left(q_1,q_2,q_3\right)dq_1dq_2dq_3=\frac{E_0}{c\rho}. \tag{28}$$

$$q_1=r\cos\gamma\cdot\cos\delta, \tag{29}$$

$$r=\sqrt{q_1^2+q_2^2+q_3^2}, \tag{30}$$

$$q_2=r\cos\gamma\cdot\sin\delta, \tag{31}$$

$$\delta = \operatorname{arctg}\frac{q_2}{q_1}, \tag{32}$$

$$q_3 = r\sin\gamma, \tag{33}$$

$$\gamma = \arcsin\frac{q_3}{\sqrt{q_1^2+q_2^2+q_3^2}}. \tag{34}$$

$$\frac{a}{r^2}\frac{d}{dr}\left(r^2\theta^\sigma\frac{d\theta}{dr}\right)+\frac{r}{3\sigma+2}\frac{d\theta}{dr}+\frac{3}{3\sigma+2}\theta=0, \tag{35}$$

$$4\pi\frac{\sqrt{k_\xi k_\eta k_\zeta}}{L^{3/2}}\int_r r^2\theta(r)\,dr=\frac{E_0}{c\rho}. \tag{36}$$

$$\theta^\sigma\cdot\theta'(0)=0. \tag{37}$$

$$\left(r^2\theta^\sigma\theta'\right)'+\frac{1}{a(3\sigma+2)}\left(\theta\cdot r^3\right)'=0, \tag{38}$$

$$4\pi\frac{\sqrt{k_\xi k_\eta k_\zeta}}{L^{3/2}}\int_r r^2\theta(r)\,dr=\frac{E_0}{c\rho}, \tag{39}$$

$$\theta^\sigma\theta'(0)=0. \tag{40}$$

$$r^2\theta^\sigma\theta'+\frac{1}{a(3\sigma+2)}r^3\theta=C_1, \tag{41}$$

$$r^2\theta^\sigma\theta'+\frac{1}{a(3\sigma+2)}r^3\theta=0; \tag{42}$$

$$4\pi\frac{\sqrt{k_\xi k_\eta k_\zeta}}{L^{3/2}}\int_r r^2\theta(r)\,dr=\frac{E_0}{c\rho}. \tag{43}$$

$$\theta(r)=\left(\frac{\sigma r_0^2}{2a(3\sigma+2)}-\frac{\sigma r^2}{2a(3\sigma+2)}\right)^{1/\sigma}. \tag{44}$$

$$4\pi\frac{\sqrt{k_\xi k_\eta k_\zeta}}{L^{3/2}}\left(\frac{\sigma}{2a(3\sigma+2)}\right)^{1/\sigma}\int_r r^2\left(r_0^2-r^2\right)^{1/\sigma}dr=\frac{E_0}{c\rho}, \tag{45}$$

$$4\pi\frac{\sqrt{k_\xi k_\eta k_\zeta}}{L^{3/2}}\left(\frac{\sigma}{2a(3\sigma+2)}\right)^{1/\sigma}r_0^{\frac{3\sigma+2}{\sigma}}\int_0^1\left(\frac{r}{r_0}\right)^2\left(1-\left(\frac{r}{r_0}\right)^2\right)^{1/\sigma}d\left(\frac{r}{r_0}\right)=\frac{E_0}{c\rho}. \tag{46}$$

$$\int_0^1 w^m \left(1-w^2\right)^p dw = \frac{\Gamma(p+1)\cdot\Gamma(m+1)/2}{2\Gamma\left(p+(m+3)/2\right)},\ \{p+1,\ m+1\}>0, \quad (47)$$

$$r_0^2(E_0) = \left[\frac{(E_0/c\rho)}{4\pi}\frac{L^{3/2}}{\sqrt{\pi k_\xi k_\eta k_\zeta}}\left(\frac{2a(3\sigma+2)}{\sigma}\right)^{1/\sigma}\frac{(3\sigma+2)(\sigma+2)}{\sigma}\frac{\Gamma\left((\sigma+2)/2\sigma\right)}{\Gamma(1/\sigma)}\right]^{\frac{2\sigma}{3\sigma+2}}. \quad (48)$$

$$T(x,y,z,t) = \frac{1}{t^{3/(3\sigma+2)}}\left(\frac{\sigma}{2a(3\sigma+2)}\right)^{1/\sigma}\times$$
$$\times\left[r_0^2 - \frac{\left(\alpha_{\xi x}x+\alpha_{\xi y}y+\alpha_{\xi z}z\right)^2\frac{L}{k_\xi}+\left(\alpha_{\eta x}x+\alpha_{\eta y}y+\alpha_{\eta z}z\right)^2\frac{L}{k_\eta}+\left(\alpha_{\zeta x}x+\alpha_{\zeta y}y+\alpha_{\zeta z}z\right)^2\frac{L}{k_\zeta}}{2t^{\frac{2}{3\sigma+2}}}\right]^{1/\sigma}. \quad (49)$$

$$r_0^2(E_0) = \frac{\left(\alpha_{\xi x}x+\alpha_{\xi y}y+\alpha_{\xi z}z\right)^2}{\left(t^{\frac{1}{3\sigma+2}}\sqrt{\frac{2k_\xi}{L}}\right)^2}+\frac{\left(\alpha_{\eta x}x+\alpha_{\eta y}y+\alpha_{\eta z}z\right)^2}{\left(t^{\frac{1}{3\sigma+2}}\sqrt{\frac{2k_\eta}{L}}\right)^2}+\frac{\left(\alpha_{\zeta x}x+\alpha_{\zeta y}y+\alpha_{\zeta z}z\right)}{\left(t^{\frac{1}{3\sigma+2}}\sqrt{\frac{2k_\zeta}{L}}\right)} \quad (50)$$

$$r_0 t^{\frac{1}{3\sigma+2}}\sqrt{\frac{2k_\xi}{L}},\ r_0 t^{\frac{1}{3\sigma+2}}\sqrt{\frac{2k_\eta}{L}},\ r_0 t^{\frac{1}{3\sigma+2}}\sqrt{\frac{2k_\zeta}{L}} \quad (51)$$

$$T(x,y,t) = \frac{1}{t^{1/(\sigma+1)}}\left(\frac{\sigma}{2a(2\sigma+2)}\right)^{\frac{1}{\sigma}}\left[r_0^2 - \frac{(x\cos\varphi+y\sin\varphi)^2\frac{L}{k_\xi}+(-x\sin\varphi+y\cos\varphi)^2\frac{L}{k_\eta}}{t^{1/(\sigma+1)}}\right]^{\frac{1}{\sigma}}, \quad (52)$$

$$r_0^2(E_0) = \left(\frac{(E_0/c\rho)}{\pi\sqrt{k_\xi k_\eta}}\cdot\frac{\sigma+1}{\sigma}\right)^{\frac{\sigma}{\sigma+1}}\left(\frac{4a(\sigma+1)}{\sigma}\right)^{1/(\sigma+1)}. \quad (53)$$

$$\sigma\in(-\infty;-1)\cup(0;\infty). \quad (54)$$

$$r_0^2(E_0) = \frac{(x\cos\varphi+y\sin\varphi)^2\frac{L}{k_\xi}+(-x\sin\varphi+y\cos\varphi)^2\frac{L}{k_\eta}}{t^{1/(\sigma+1)}}, \quad (55)$$

$$l_\xi(t) = r_0 t^{\frac{1}{2\sigma+2}}\sqrt{\frac{k_\xi}{L}};\ l_\eta(t) = r_0 t^{\frac{1}{2\sigma+2}}\sqrt{\frac{k_\eta}{L}}. \quad (56)$$

$$x = r_0 t^{\frac{1}{2\sigma+2}} \left(\frac{L}{k_\xi} \cos^2 \varphi + \frac{L}{k_\eta} \sin^2 \varphi \right)^{-1/2}, \tag{57}$$

$$y = r_0 t^{\frac{1}{2\sigma+2}} \left(\frac{L}{k_\xi} \sin^2 \varphi + \frac{L}{k_\eta} \cos^2 \varphi \right)^{-1/2}, \tag{58}$$

$$v_x = \frac{dx}{dt} = \frac{r_0}{2\sigma+2} \cdot \left(\frac{L}{k_\xi} \cos^2 \varphi + \frac{L}{k_\eta} \sin^2 \varphi \right)^{-1/2} t^{-\frac{2\sigma+1}{2\sigma+2}}, \tag{59}$$

$$v_y = \frac{dy}{dt} = \frac{r_0}{2\sigma+2} \cdot \left(\frac{L}{k_\xi} \sin^2 \varphi + \frac{L}{k_\eta} \cos^2 \varphi \right)^{-1/2} t^{-\frac{2\sigma+1}{2\sigma+2}}. \tag{60}$$

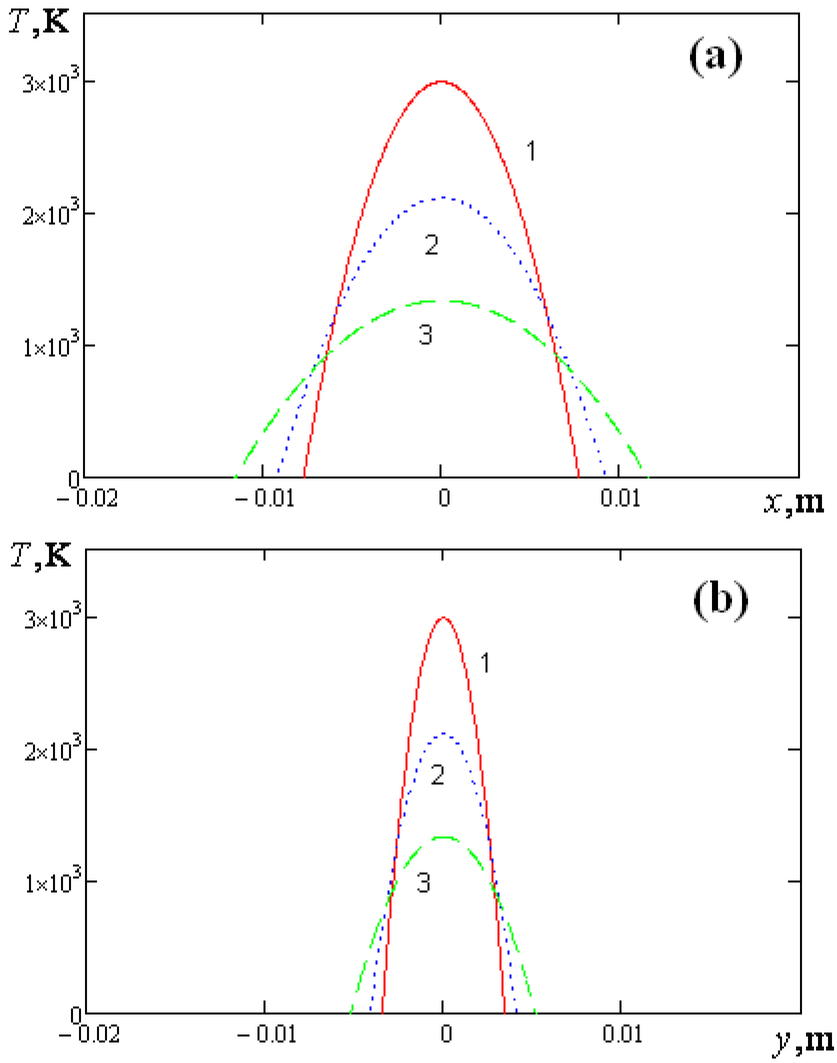

Figure 1. *Temperature and temperature front of the thermal wave for* $\sigma = 1$ *at moments of time 1 –* $t = 10^{-6}$ s, *2 –* $t = 2 \cdot 10^{-6}$ s, *3 –* $t = 5 \cdot 10^{-6}$ s, *for the cross sections (a) –* $y = 0$, *(b) –* $x = 0$.

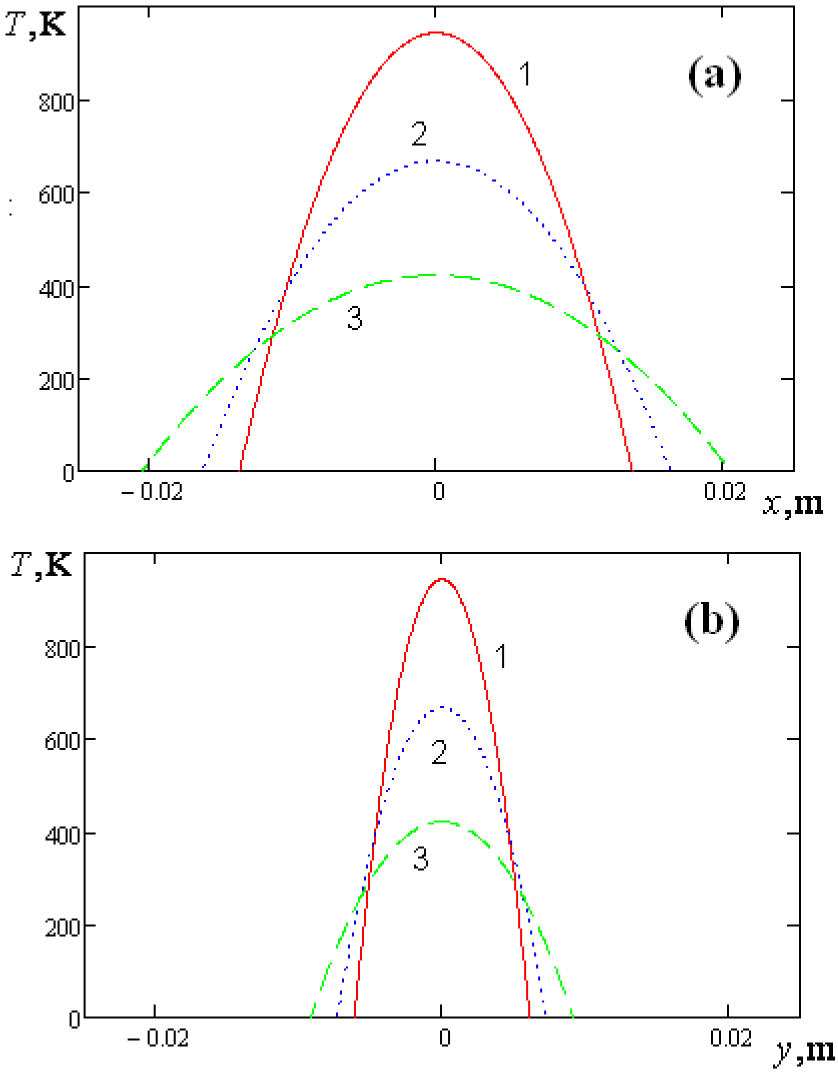

Figure 2. *Temperature and temperature front of the thermal wave for* $\sigma = 1$ *at moments of time 1 –* $t = 10^{-5}$ s, *2 –* $t = 2 \cdot 10^{-5}$ s, *3 –* $t = 5 \cdot 10^{-5}$ s, *for the cross sections (a)* $- y = 0$, *(b)* $- x = 0$.

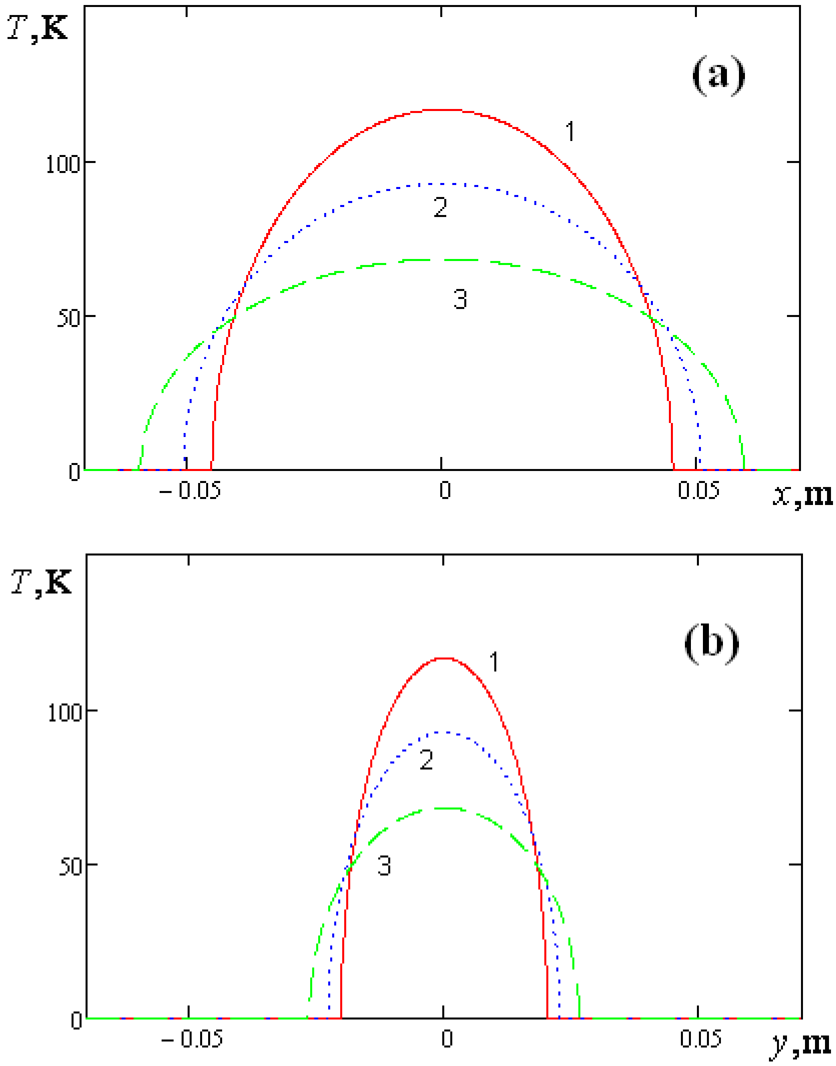

Figure 3. *Temperature and temperature front of the thermal wave for* $\sigma = 1$ *at moments of time 1 –* $t = 2\,\mathrm{s}$*, 2 –* $t = 4\,\mathrm{s}$*, 3 –* $t = 8\,\mathrm{s}$*, for the cross sections (a) –* $y = 0$*, (b) –* $x = 0$.