COMPORTAMENTO DINÂMICO DE UMA PLACA FIXADA SOB CARGA MÓVEL

DYNAMICS OF A CLAMPED RIBBED PLATE UNDER MOVING LOADS

ДИНАМИЧЕСКОЕ ПОВЕДЕНИЕ ЗАЩЕМЛЕННОЙ ПЛАСТИНЫ ПОД ДЕЙСТВИЕМ ПОДВИЖНОЙ НАГРУЗКИ

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RESUMO

Na formulação quase-estática, resolve-se o problema do comportamento dinâmico de uma placa fina elástica discretamente suportada por um sistema de rigidificadores, sobre a qual se move uma carga distribuída linear infinita. A todos os lados da placa são fixados. A solução é baseada no método de Bubnov-Galerkin. Como resultado, a equação diferencial parcial reduz-se a um sistema de equações diferenciais ordinárias para funções desconhecidas do tempo. Acredita-se que a placa sob carga móvel esteja num modo quase-estático, o que corresponde à sua superfície curva constante no tempo. Ao resolver um problema na formulação dinâmica, assume-se que a superfície curva da placa sob carga móvel muda não apenas em termos de coordenadas espaciais, mas também no tempo. As velocidades críticas de seu movimento são determinadas. Com base na pesquisa realizada, a possibilidade de reduzir os valores máximos de acelerações em determinados pontos de estruturas de armação com rigidificadores para lançadores de sistemas aeroespaciais foi revelada. Os exemplos são considerados.

Palavras-chave: *placa fina, solução quase-estática, velocidades críticas, rigidificadores.*

ABSTRACT

In the quasistatic formulation, the problem of the dynamic behavior of a thin elastic discretely reinforced system of plate stiffeners is solved, on the surface of which an infinite linear distributed load moves. The plate is clamped from all sides. The solution is based on the Bubnov-Galerkin method. As a result, the partial differential equation reduces to a system of ordinary differential equations with respect to unknown time functions. It is believed that the plate under the action of a movable load is in a quasistatic regime, to which there corresponds a constant in time its curved surface. When solving a problem in a dynamic formulation, it is considered that the curved surface of the plate under the action of a moving load changes not only in spaced coordinates, but also in time. The critical velocities of its motion are determined. On the basis of the conducted researches, the possibility of reducing the maximum values of accelerations at given points of frame structures with stiffeners for

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launching installations of aerospace systems has been revealed. Examples are considered.

Keywords: *thin plate, quasistatic solution, critical velocities, stiffeners.*

АННОТАЦИЯ

В квазистатической постановке решена задача о динамическом поведении тонкой упругой дискретно подкрепленной системой ребер жесткости пластины, по поверхности которой движется бесконечная погонная распределенная нагрузка. Пластина защемлена со всех сторон. Решение проводится на основе метода Бубнова-Галеркина. В результате уравнение в частных производных сводится к системе обыкновенных дифференциальных уравнений относительно неизвестных функций времени. Считается, что пластина под действием подвижной нагрузки находится в квазистатическом режиме, которому соответствует неизменная во времени ее изогнутая поверхность. При решении задачи в динамической постановке считается, что изогнутая поверхность пластины под действием подвижной нагрузки изменяется не только по пространственным координатам, но и во времени. Определены критические скорости ее движения. На основе проведенных исследований выявлена возможность снижения максимальных значений ускорений в заданных точках каркасных конструкций с ребрами жесткости для стартовых установок авиакосмических систем. Рассмотрены примеры.

Ключевые слова: *тонкая пластина, квазистатическое решение, критические скорости, ребра жесткости.*

INTRODUCTION

The thin-walled structures, reinforced with stiffeners, are widely used in engineering practice and are exposed to various dynamic loads. Reinforcements change the frequency spectrum and the mode of plate vibrations (Yakushev, 1990; Zhigalko and Dmitrieva, 1978; Bulatova and Siniakova, 2017). It should be noted that the stiffeners slightly increase the weight of the structure, but significantly increase its strength and are indispensable for the transfer of forces close to concentrated. When solving dynamic problems for plates reinforced with stiffeners, it becomes necessary to take into account their discrete arrangement (Klimanov and Logginskaya, 1974; Vlasov, 1967; Gurevich, 1972; Zhestkii, 1972).

Stiffened plates are one of the main structural elements of thin-walled systems of aircraft, missile and other models of equipment under the action of a movable load. Because of this, the problems of the action of movable loads on stiffened plates have quite definite technical applications (Zhao *et al.*, 2018; Kotlikov *et al.*, 2018; Souad and Brahim, 2018; Asjad *et al.*, 2016; Hassan and Chatterjee, 2015). A review of the works on this subject is given in (Konoplev and Yakushev, 2003; Medvedsky and Rabinsky, 2007), but the corresponding problems are considered there mainly for smooth plates. Stiffened plates, and moreover, taking into account the discreteness of the arrangement of the stiffeners in them were not considered (Kachur and Sumin, 1973; Kiselevskaya, 1962; Kovinskaya and Nikiforov, 1973; Moshensky, 1955; Osipova and Fleishman, 1973; Postnov, 1965; Hanzhov, 1970; Berger, 1970). In the present paper, in quasistatic and dynamic formulations, the problem of the dynamic behavior of plates is solved, taking into account the discrete arrangement of the stiffeners (stringers). In this case, in contrast to (Antufev *et al.*, 2017), the stiffened plate with a rigid fixing is considered here. Previously, this type of work was presented in (Volmir, 1967).

Let us consider a thin elastic plate of rectangular shape in plan, referred to the Cartesian coordinate system *0xyz.* On its surface in the direction of the *x*-axis at a constant velocity *V,* an infinite uniformly distributed over the area of the plate inertial load of intensity *p* moves, which is conditionally shown in the form of distributed forces along a line perpendicular to the *x*-axis (Figure 1).

CHARACTERISTICS OF THE STIFFENED PLATE

The authors consider a thin elastic plate of rectangular shape in plan, referred to the Cartesian coordinate system *0xyz.* On its surface in the direction of the *x*-axis at a constant velocity *V,* an infinite uniformly distributed over the area of the plate inertial load of intensity *p* moves, which is conditionally shown in the form of distributed forces along a line perpendicular to the *x*-axis (Figure 1).

In the same figure, the dotted lines parallel to the *x*-axis also show the stringers that support it. In the future, the inertia of plate motion is determined mainly by the dynamic action of centrifugal forces of inertia, and its mass does not play an important role (Panovko and Gubanova, 1977). For a smooth plate, we solve the problem in quasistatic and dynamic formulations in order to choose the optimal variant for investigating the dynamic behavior of the already stiffened plate (Formalev and Kolesnik, 2017; Formalev and Kolesnik, 2018; Roux *et al.*, 2015; Gibigaye *et al.*, 2016).

The authors believe that the plate under the action of the movable load is in a quasistatic mode, which corresponds to the time constant *t* its curved surface *w(x,y).* At the same time, it is also the surface of motion of the load elements, which in a time *t* pass through the distance *x* = *Vt*. Due to the curvature of the curved plate surface, the force acting on it is determined by the sum of the weight of the linear load *p* and its inertia force $(p/g)\cdot\partial^2 w/\partial t^2$. Without taking into account the weight of the plate, the intensity of the total linear load, taking into account the relation *x* = *Vt,* is Equation 1, where *g* – gravitational acceleration. Then the bending equation of the plate in the quasistatic mode takes the following form Equation 2. Here $D = Eh^3/(1-\nu^2)$ – cylindrical rigidity of the plate; *h, E* and *ν* – thickness, modulus of elasticity and Poisson's ratio of its material, respectively, ∇^2 – Laplace operator. To solve the partial differential Equation 2, we use the Bubnov-Galerkin method, in accordance with which we represent the deflection of *w* as a factorization (Equation 3). Here w_{mn} – unknown coefficients, $\varphi_{mn}(x,y)$ – orthogonal forms of natural vibrations of the plate. After applying the procedure of the Bubnov method, the differential Equation 2 goes into the system of $K\times L$ linear algebraic equations with respect to the coefficients w_{mn}. However, since the eigenform of the vibrations are orthogonal $\varphi_{mn}(x,y)$, this system breaks down into separate, unrelated equations for each pair of values of *m* and *n*. The solution of each of them has the form Equation 4, where *S* – area of the plate. When the denominator of formula (4) tends to zero, the deflection of the plate increases indefinitely. From this condition, it is possible to determine the square of the critical velocity of the load V_{KP}^2 in form $m\times n$ (Equation 5). When performing concrete calculations, the integral in the denominator of formula (5) will be negative and therefore V_{KP}^2 will become positive.

SOLVING THE PROBLEM IN A DYNAMIC FORMULATION:

When solving the problem in a dynamic formulation, we assume that the curved surface of the plate under the action of a movable load of intensity *p* varies not only with respect to the spaced coordinates *x* and *y* but also in time *t.* In this case, the projection of the velocity of an element of a uniformly moving load *p* on the vertical axis of the plate will be equal to the already full derivative and the vertical acceleration of this element, taking into account the relation *x* = *Vt,* is Equation 6. The second term in Equation 6 containing the mixed derivative corresponds to the Coriolis acceleration and is usually neglected in solving practical problems. Then the equation of plate motion under the action of the gravitational and inertial components of the load will be Equation 7. For its approximate solution, we also apply the Bubnov method, in accordance with which we represent the deflection of the plate as a factorization (Equation 8), where $w_{mn}(t)$ – unknown functions of time, $\varphi_{mn}(x,y)$ – forms of natural vibrations of the plate. Substituting the factorization Equation 8 into Equation 7 and applying the Bubnov-Galerkin method to the latter, we reduce it to the system $K\times L$ of differential equations of the second order but already in ordinary derivatives. However, due to the orthogonality of the eigenmodes of the plate, $\varphi_{mn}(x,y)$ it splits into separate for each pair of *m* and *n* non-related differential equations of the form Equation 9, where the points of the function *w* denote its time derivatives, and ω_{mn}^2 – square of the natural vibrations frequency of the plate in form $m\times n$. The coefficients entering into Equation 9 have the following form Equations 10, 11. The standard Equation 9 for given initial conditions has a solution in closed form. In order to determine the critical velocity of the load, we use the dynamic stability criterion. According to which, in the critical state, the natural frequencies of the system's vibrations go to zero ($\omega_{mn} = 0$). From this condition, we define the square of the critical velocity of the load in the form $m \times n$. It completely coincides with formula (5) obtained under the assumption of quasistatic deformation of the plate. Therefore, in the future, when solving problems about the motion of the load over discretely supported plates, we will use a simpler quasistatic approach.

Let the plate be discretely supported by a system of elastic stringers. In solving the problem, we assume that the neutral line of these stiffeners lies in the middle surface of the plate. Therefore, they can be considered as one-dimensional elastic inclusions. To solve the problem, we separate the stringers from the plate mentally and replace their effect distributed along the contact lines of the bodies $y = y_i$ in the middle surface of the plate by interaction reactions $q_i(x)$ directed along the *z*-axis. Since the stiffness of the plate in the tangential directions *x* and *y* is much larger than in the direction of the normal to its surface, then we neglect the tangential reactions of the contact. Then, under the assumption of a quasistatic character of plate deformation, the equation of its bending becomes Equation 12, where *C* – the number of stiffeners, and delta – Dirac function $\delta(y - y_i)$ determine the coordinates of the location of stringers along the *y*-axis. Each of the stringers is assigned to a rectangular coordinate system *0xz* (Figure 1). Equations of their equilibrium in the projection onto the *z*-axis have the form Equation 13. Here EJ_i and z_i – their flexural rigidity and deflections, respectively. The mass of the stringer itself, as well as the inertia of its movement, by analogy with the plate, is neglected. On each of the contact lines the condition of equality of deflections of both bodies $w(x, y_i) = z_i$, but we assume that the deformed states caused by neighboring contact reactions do not interfere with each other. Then, substituting the reaction of the contact q_i from (12) into Equation 11 we obtain the resolving equation of the problem in the form Equation 14. It is a partial differential equation with discontinuous coefficients in the direction of the y-axis, which is due to the presence in the last term of its left side of the delta – the Dirac functions. To solve it, we use the Bubnov method, in accordance with which we represent the deflection of the *w* plate also in the form (3). Applying the procedure of the Bubnov method to Equation 13, we reduce it to the coupled system $K \times L$ of linear algebraic equations with respect to the coefficients w_{mn} in the factorizations (3). In the matrix form of the record, it has the form Equation 15, where Equation 16. The dimension of the stiffness matrix *K*, the inertia *M* and the vectors *W, F* is determined by the number of terms of the series stored in the factorization (3). The elements of the matrices *K, M,* and the vector *F* have the form Equations 17,18,19. The matrix *K* has a block-diagonal character, which is due to the presence of a discontinuity in its coefficients only along the *y*-axis. In each of the blocks, its first term is diagonal. When solving the system of Equations 13 with changing load velocities *V*, the deflections of the plate will also change. If they begin to increase sharply for some values of *V*, this means that we are approaching the critical behavior.

EXAMPLES OF EXPERIMENTAL STUDIES:

As a first example, let us consider a smooth plate rigidly clamped around the edges. Then the function $\varphi_{mn}(x, y)$ in the Equation 3 and Equation 8 we take in the form Equation 20, where (*m=1,2,..K;n=1,3,….L*)

Substituting Equation 19 into the Equation 5 and carrying out the necessary calculations for the simplest form of stability loss with wave numbers *m=n=1,* we obtain the minimum value of the critical velocity similar to that of (Antufev, 2017).

With this in mind, after a number of transformations, the quasidynamic deflections of the plate center (*x = y = 0*), on the ground of the Equation 4, can also be written in the form Equations 21, 22.

Where V_{KP}^2 – square of the critical speed of the movable load, determined by the Equation 4, w_{CT} – static deflection of the center of the plate. For a plate, with a relative thickness $a/h = 20$ dimensionless static deflection $w_{CT}^* = w_{CT}(p/Eh)$ will be $w_{CT}^* = 1{,}168 \cdot 10^{-5}$, and in the monograph (Timoshenko, 1948) the same value is $w_{CT}^* = 1{,}136 \cdot 10^{-5}$. Thus, we can assume that the use of a monomial approximation of unknowns in the solution of this problem is justified. Figure 2 shows the dependence of the dimensionless deflection of the center of the plate $w^* \cdot 10^5$ from the square of the relative velocity of the load V^2 / V_{KP}^2. When $V \to V_{KP}$ deflections increase without limit. The dashed line shows the dependence of the dimensionless deflection of the center of the plate on the square of the relative velocity for the hinged plate, given in (Antufev *et al.*, 2017). For a plate with one stringer, in the monomial approximation, further neglecting the gravitational component of the load, the square of the critical velocity of its motion will be $V_{KP}^2 = k_{11} / m_{11}$, where the stiffness and inertia coefficients are calculated from the Equation 16 (Equation 23). The first term in this formula coincides with the value of the square of the critical velocity for the smooth plate (Equation 5). Consequently, the presence of even one stringer leads to a sharp increase in the critical load velocity.

CONCLUSIONS:

The dynamic behavior of stiffened plates is devoted to a sufficient number of works, the main ones of which can be considered the following, but there are not so many works related to the action of the movable load. In this paper, we studied the dynamic behavior of a rigidly clamped plate reinforced with stiffeners under the action of the movable load. It is shown that the use of a monomial approximation of unknowns in solving such problems is justified. Determined that the dependence of the dimensionless deflection of the plate center on the square of the relative velocity for a rigidly clamped plate differs significantly from the analogous dependence of the hinged plate.

It is determined that the nature of the deformation of a cylindrical shell under non-axisymmetric high-speed loading significantly depends on the rate of load application. Under quasistatic loading, the characteristic manifestation of nonlinearity of the inhomogeneous stress-strain state of the shell is a smooth restructuring of the shape of the bend in the loading process, while preserving the known behavior patterns of the shells under static loading depending on the degree of non-uniformity of the load. Dynamic loading is characterized by the maximum sensitivity of the structure to the magnitude of the impact velocity, the change in the parameters of the structure and the localization of the pressure profile. It has been established that during high-speed loading it remains during the entire time of deformation of the similarity of the flexural form, fixed by plastic deformations. This type of deformation is distinguished by the insensitivity of critical loads to the degree of heterogeneity, the approximation (with increasing degree of heterogeneity) of the level of critical loads to a certain asymptote, a significant increase, compared to static loading, the level of critical pressures.

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$$p - \frac{p}{g}\frac{\partial^2 w}{\partial t^2} = p - \frac{p}{g}V^2\frac{\partial^2 w}{\partial x^2} \tag{1}$$

$$D\nabla^2\nabla^2 w = p - \frac{p}{g}V^2\frac{\partial^2 w}{\partial x^2}. \tag{2}$$

$$w = \sum_m^K \sum_n^L w_{mn}\varphi_{mn}(x, y) \tag{3}$$

$$w_{mn} = \frac{p\int\limits_S \varphi_{mn} dS}{D\int\limits_S \nabla^2\nabla^2\varphi_{mn}\cdot\varphi_{mn} dS + \frac{p}{g}V^2\int\limits_S \frac{\partial^2\varphi_{mn}}{\partial x^2}\varphi_{mn} dS} \tag{4}$$

$$V_{KPmn}^2 = -\frac{Dg\int\limits_S \nabla^2\nabla^2\varphi_{mn}\cdot\varphi_{mn} dS}{p\int\limits_S \frac{\partial^2\varphi_{mn}}{\partial x^2}\varphi_{mn} dS}. \tag{5}$$

$$\frac{d^2 w}{dt^2} = \frac{d}{dt}(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}) = \frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial t\partial x} + V^2\frac{\partial^2 w}{\partial x^2} \tag{6}$$

$$D\nabla^2\nabla^2 w = p - \frac{p}{g}(\frac{\partial^2 w}{\partial t^2} + V^2\frac{\partial^2 w}{\partial x^2}), \tag{7}$$

$$w = \sum_m^K \sum_n^L w_{mn}(t)\varphi_{mn}(x, y), \tag{8}$$

$$\ddot{w}_{mn} + \omega_{mn}^2 w_{mn} = f_{mn} \quad (m = 1,2,...K; n = 1,2,...L), \tag{9}$$

$$\omega_{mn}^2 = \frac{D\int\limits_S \nabla^2\nabla^2\varphi_{mn}\cdot\varphi_{mn} dS + V^2\frac{p}{g}\int\limits_S \frac{\partial^2\varphi_{mn}}{\partial x^2}\varphi_{mn} dS}{\frac{p}{g}\int\limits_S \varphi_{mn}^2 dS}, \tag{10}$$

$$f_{mn} = \frac{g\int\limits_S \varphi_{mn} dS}{\int\limits_S \varphi_{mn}^2 dS}. \tag{11}$$

$$D\nabla^2\nabla^2 w = p - \frac{p}{g}V^2\frac{\partial^2 w}{\partial x^2} - \sum_i^C q_i\delta(y - y_i)\,, \quad (12)$$

$$EJ_i\frac{d^4 z_i}{dx^4} = q_i\,. \text{ (i = 1,2, ... C)} \quad (13)$$

$$D\nabla^2\nabla^2 w + \frac{p}{g}V^2\frac{\partial^2 w}{\partial x^2} + \sum_i^C\left(EJ_i\frac{d^4 z_i}{dx^4}\right)\delta(y - y_i) = p\,. \quad (14)$$

$$\left[[K] - V^2[M]\right]\{W\} = \{F\}, \quad (15)$$

$$K = \left[k_{mn}^{kl}\right], \quad M = \left[m_{mn}^{kl}\right], \quad W = \{w_{kl}\}, \quad F = \{f_{kl}\}. \quad (16)$$

$$k_{mn}^{kl} = D\int_S \nabla^2\nabla^2\varphi_{mn}\cdot\varphi_{kl}dS + \sum_{i=1}^C EJ_i\int_S \delta(y - y_i)\frac{\partial^4\varphi_{mn}}{\partial x^4}\varphi_{kl}dS,. \quad (17)$$

$$m_{mn}^{kl} = \frac{p}{g}\left|\int_S \frac{\partial^2\varphi_{mn}}{\partial x^2}\varphi_{kl}dS\right|, \quad (18)$$

$$f_{kl} = p\int_S \varphi_{kl}dS\,. \quad (19)$$

$$\phi_{mn}(x, y) = \sin^2\frac{m\pi x}{l}\cos^2\frac{n\pi y}{2a}. \quad (20)$$

$$w = w_{CT}\frac{1}{1 - V^2/V_{KP}^2}, \quad (21)$$

$$w_{CT} = \frac{ph\int_S \varphi dS}{D\int_S \nabla^2\nabla^2(\varphi)\varphi dS}, \quad (22)$$

$$V_{CR}^2 = -\frac{Dg}{p}\frac{\int_S \nabla^2\nabla^2\varphi\cdot\varphi dS + EJ\int_S \delta(y - 0)\frac{\partial^4\varphi}{\partial x^4}\varphi dS}{\int_S \frac{\partial^2\varphi}{\partial x^2}\varphi dS}. \quad (23)$$

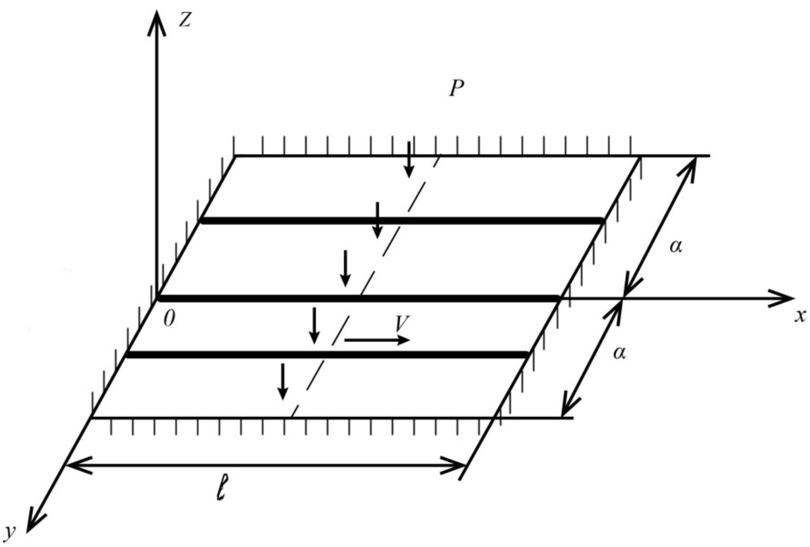

Figure 1. *Conditional scheme of the clamped plate and the inertial load of intensity*

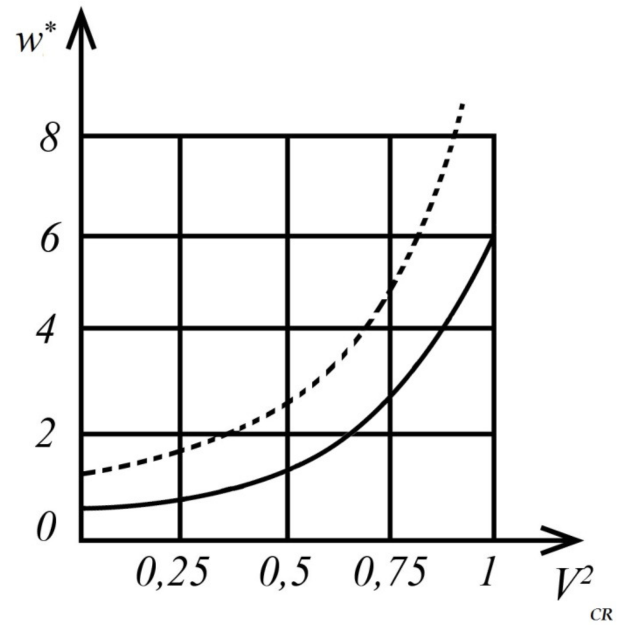

Figure 2. *Dependence of the dimensionless deflection of the plate center*