MODELAGEM MATEMÁTICA DE PROCESSOS DINÂMICOS EM SISTEMAS MECÂNICOS NÃO LINEARES EXISTENTES COM PARÂMETROS CONCENTRADOS

MATHEMATICAL MODELING OF DYNAMIC PROCESSES IN ESSENTIALLY NONLINEAR MECHANICAL SYSTEMS WITH LUMPED PARAMETERS

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДИНАМИЧЕСКИХ ПРОЦЕССОВ В СУЩЕСТВУЮЩИХ НЕЛИНЕЙНЫХ МЕХАНИЧЕСКИХ СИСТЕМАХ С СОСРЕДОТОЧЕНЫМИ ПАРАМЕТРАМИ

KONDRATENKO, Leonid A.[1*]; MIRONOVA, Lyubov I.[2]; DMITRIEV, Vladimir G.[3]; KUZNETSOVA, Elena L.[4]

[1,3] Moscow Aviation Institute (State National Research University), Department of Machine Science and Machine Components, 4 Volokolamskoe shosse, zip code 125993, Moscow – Russian Federation
(phone: +7 499 158-40-43)

[2]National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Department of Power Engineering, 31 Kashirskoe shosse, zip code 115409, Moscow – Russian Federation
(phone: +7 495 788-56-99)

[4] Moscow Aviation Institute (State National Research University) 4 Volokolamskoe shosse, zip code 125993, Moscow – Russian Federation
(phone: +7 499 158-40-43)

** Corresponding author*
e-mail: kondrat.leonid@yandex.ru

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RESUMO

O artigo apresenta um sistema de equações diferenciais que descreve os processos de transferência de movimento em mecanismos, levando em conta não-linearidades essenciais causadas por fenômenos de histerese, zonas mortas, limitações de pressão, etc. O algoritmo computacional é baseado no método Runge-Kutta de quarta ordem. A solução de um sistema de equações diferenciais ordinárias é descrita usando não-linearidades essenciais. Os autores, em cada etapa do cálculo, levaram em conta as peculiaridades das não-linearidades presentes no sistema. Os resultados da modelagem da mudança na velocidade de rotação do elemento de saída e as tensões nas linhas mecânicas após um aumento gradual na ação de controle são apresentados. A modelagem foi realizada utilizando um computador com etapas de contagem de $H=10^{-4}$; $2*10^{-4}$s. O erro não excedeu 10%.

Palavras-chave: *equações diferenciais, não-linearidades essenciais, mecanismo, velocidade de rotação, processo transitório.*

ABSTRACT

The paper presents a system of differential equations that describe the processes of motion transfer in mechanisms taking into account the essential nonlinearities caused by hysteresis phenomena, dead zones, pressure limitations, etc. The computational algorithm is based on the Runge-Kutta method of the fourth order. A solution to a system of ordinary differential equations is described in the presence of essential nonlinearities. Here, at each stage of the calculation, the features of the nonlinearities that are present in the system are taken into account. The results of modeling the change in the speed of rotation of the output link and stresses in mechanical lines after a stepwise increase in the control action are given. The simulation was carried out with the help of a computer with the steps of counting $H=10^{-4}$; $2*10^{-4}$s. The error did not exceed 10%.

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Keywords: *differential equations, essential nonlinearities, mechanism, rotational speed, transient process.*

АННОТАЦИЯ

В работе представлена система дифференциальных уравнений, которая описывает процессы переноса движения в механизмах с учетом существенных нелинейностей, вызванных явлениями гистерезиса, мертвыми зонами, ограничениями давления и т.д. Вычислительный алгоритм основан на методе Рунге-Кутты четвертого порядка. Решение системы обыкновенных дифференциальных уравнений описано с использованием существенных нелинейностей. Авторами, на каждом этапе расчета, учтены особенности нелинейностей, которые присутствуют в системе. Представлены результаты моделирования изменения скорости вращения выходного звена и напряжений в механических линиях после ступенчатого увеличения управляющего воздействия. Моделирование проводилось с помощью компьютера со ступенями подсчета H=10^{-4}; 2*10^{-4} сек. Ошибка не превышала 10%.

Ключевые слова: *дифференциальные уравнения, существенные нелинейности, механизм, скорость вращения, переходный процесс.*

INTRODUCTION

Modern industry is a set of diverse industries, an essential role in which are the mechanical systems that are an integral part of machines for the conversion of energy and (or) movement. Thanks to the use of such devices, labor productivity increases, physical and mental labor of a person are facilitated (Kuznetsova *et al.*, 2018). A mechanical system or mechanism, being the carrier of certain motions, is a set of interrelated bodies that transform the movements of some bodies into the required motions of others. Most of these mechanisms have moving links (Rizzatti *et al.*, 2017). When operating, these links either perform useful work or transmit the energy of the movement to accomplish such work. In all cases, deformation occurs and stresses appear in the links that counterbalance external influences (Lomakin *et al.*, 2017). As a consequence, it becomes necessary to calculate the velocities and voltages at the points of the cross sections when power is transferred from the input to the output (Kondratenko, 2008).

Modern methods for investigating the dynamics of mechanisms are based on Lagrange's equation of the second kind and Helmholtz's theorem, on the basis of which a well-known wave equation describing the movement of particles of matter is derived (Sedov, 1970). With the use of these approaches, the practical problems of studying the oscillations of the velocities of motion and stresses are solved indirectly. Here, there is a need to search for changes in the second derivative of displacement and in the recalculation of displacement to stress.

If the control system is considered, then the subject of research is often the quality of management (error, system performance, stability). To solve such problems, many methods have been developed (Besekerskij and Popov, 1966; Voronov *et al.*, 1977). The quantitative characteristics of the oscillations of the output and intermediate coordinates of the system (speed of motion, voltage) are usually not studied, although it is these coordinates that often play a decisive role in the effectiveness of using a particular design. Therefore, the methods used for mathematical modeling of dynamic processes of motion transfer in mechanical systems should be based on a combined analysis of the problems of mechanics and control theory. The theoretical aspects of these disciplines adequately describe dynamic processes in mechanical systems. The following approach is suggested.

MATERIALS AND METHODS

From the point of view of mechanics, the mathematical model sought will encompass the class of symbolic mathematical objects in the form of numbers and vectors (Korn and Korn, 1968), since the parameters of the motion transfer system can be described by generalized coordinates in a multidimensional space (Lanczos, 1962). This approach involves performing mathematical modeling in a linear vector space using methods of linear or vector algebra. However, real mechanisms contain various nonlinearities, including essential nonlinearities (friction, hysteresis, dead band, pressure limitation, distributed parameters, etc.). Dynamic properties and the form of the static characteristics of the object introduce distortions into the process of functioning, thus affecting the output of mechanical characteristics. This system can not be described only by linear dependencies. Analytic studies of such systems are extremely difficult.

Taking into account Hamilton's formalism, the dynamic characteristics of an object are, as a rule, described by integrable differential relations that relate the speed of motion of a body point to a force action under equilibrium conditions. Such relations reduce to systems of linear and nonlinear differential equations. The search for solutions of such systems is also connected with mathematical difficulties.

If the problem of finding all solutions of a differential equation can be reduced to a finite number of algebraic operations, operations of integration and differentiation of known functions, then we say that the equation is integrated into quadratures. In particular, for the determination of stresses, in applications of the theory of elasticity, it is extremely rare to find equations that are integrable in quadratures (Mironova, 2013). Taking into account the fact that the mathematical model should include systems of differential equations with essential and nonessential nonlinearities, mathematical modeling should be carried out using numerical methods.

This approach was carried out on the basis of research of dynamic hydraulic drive processes and is described in the next section. First, the process of linear hydraulic drive functioning was considered, and then nonlinearities were taken into account.

RESULTS AND DISCUSSION:

3.1. The theoretical aspect of the dynamics of the mechanical system

In many cases, the task of the mechanism is to maintain some dynamic regime that ensures maximum efficiency of the executive body with the greatest resource of all the links involved in the work. Since in this case, it is necessary to consider the cases of deviation of the rotational speed (speed of motion) from a certain steady-state value, which ensures a rational operating mode of the mechanism, then in accordance with the theory of stability (Babakov, 1958), it becomes necessary to study equations of the form (Equations 1, 2). Here ζ_i i are the coordinates of the system; Ω – the speed of the technological object. Then the investigation reduces to solving Equation 3.

From the point of view of control theory, such a path seems natural. In mechanics, however, the problems of determining the change in coordinates and the shape of oscillations are often solved (Grigolyuk and Selezov, 1973; Guz' and Babich, 1985; Lur'e, 1950). Such processes are usually studied by the methods of elasticity theory, for example, using the Equations 4, 5 (Babakov, 1958). Whose solution is sought in the form Equation 6. In the Expressions 4 – 6 the following notations are accepted: ν, E - mass characteristics and elastic characteristic of the mechanical line; f – cross-sectional area; Q – the intensity of the external load; H, θ, p, α – constants determined from the initial conditions.

With the use of the Lagrange equation of the second kind, oscillations of the kinetic and potential energies of motion of a mechanical system are considered Equations 7, 8. Where i = 1, 2, …, s. Here T, U – kinetic and potential energies; q_i – the generalized coordinate and its derivative; Q_i – generalized force. The parameters of motion of the mechanism from this equation are determined after some transformations since the Euler-Lagrange formalism is based on the energy approach and scalar quantities in the configuration space (Lanczos, 1962). An example of solving one of such problems is given in (Kondratenko *et al.*, 2017a).

Equations 1-8, which underlie many works on the dynamics of machines, allow us to estimate the change in the displacement of a part section in time and space under given boundary conditions. However, such information, when it is necessary to account for the interconnection of a large number of factors, is often excessive. To assess the system's performance, it is usually sufficient to know under what conditions auto-oscillations of the rotation frequency of a technological object (executive body) occur, i.e. stability is lost, and under what conditions stability is not lost. At the same time, because of the lack of information about the developed stresses caused by the dynamics of motion, it is difficult to estimate the probability of the destruction of the detail or the violation of the working process (Kondratenko, 2005). The above approach does not allow us to take into account explicitly the stress changes and rheological concepts (Reiner, 1958) on the transfer of motion in mechanical systems.

In articles (Kondratenko *et al.*, 2017b; Kondratenko, 2017; Kondratenko and Mironova, 2018), the dynamic characteristics of mechanisms with elastic mechanical lines were studied taking into account these important factors (rheology and elastic response of the system). The Simulation models correctly reflected the processes of motion transfer.

Let us demonstrate the essence of modeling dynamic processes by the example of the functioning of a hydrostatic power drive, using other approaches.

3.2. Modeling the transmission of motion in a volumetric hydraulic drive

The volumetric hydraulic drive is widely used in machine building, machine-tool construction, mining, and construction industries. In Figure 1 is a block diagram of a volumetric hydraulic drive. The control of the speed of rotation of the actuator is performed by changing the flow rate of the liquid supplied by the pump. The process of transferring motion in it can be represented by the structural diagram of Figure 2. The considered mechanism contains essentially nonlinear elements: backlash, relay elements with dead band and hysteresis loop, safety valves, etc.. In this case, the motion transfer process is accompanied by various nonlinear effects such as electromagnetic phenomena, friction in the load, isothermal and adiabatic processes in the working fluid, and the like. The distributed parameters of the system further complicate the decision and make it vague. The dynamics of such a system can not already be described only by linear differential equations.

In the absence of significant nonlinearities and neglect of rheological constants, the dynamic process of motion transfer can be described by a system of Equations 9 – 15 (Kondratenko, 2008; Kondratenko *et al.*, 2017c) (Figure 3). Here p_{p1}, p_{h1} – fluid pressure in the pumping main at the pump and the hydraulic motor; p_{p2}, p_{h2} – fluid pressure in the filling shunt line at the pump and hydraulic motor; υ_{p1}, υ_{h1} – velocity of the liquid in the pressure pipeline in the section adjacent to the pump and the hydraulic motor; υ_{p2}, υ_{h2} – velocity of the fluid in the filling shunt line in the section adjacent to the pump and the hydraulic motor; f_p, f_d, V_p, V_d – cross-sectional areas and volumes of the discharge line and filling shunt line; w_p, w_h - volumetric constants of the pump and hydraulic motor; τ – criterion of tightness of the hydraulic drive; ϑ_p, ϑ_d – elasticity of pressure line and filling shunt line; K – coefficient of leakage of working fluid in the pump; κ – modulus of elasticity of working fluid; J – flywheel moment of inertia of the executive body; c – coefficient of friction loss proportional to the pressure drop; h – coefficient of friction loss proportional to the speed of motion; Ω_1, Ω_2 – angular speeds of rotation of the pump shaft and hydraulic motor shaft.

Applying numerical methods (Butcher, 2008; Iserles, 1996; Kireev and Panteleev, 2015), for example, the method of successive approximations, one can obtain a solution of the Cauchy problem for any differential equation (linear or nonlinear) or for a system of equations satisfying the conditions of the existence and uniqueness theorem. However, the presence of essential nonlinearities (N_1, N_2, N_3, Figure 3) makes the task much more difficult. The study of systems of equations with one essential nonlinearity can be relatively easily solved using the method of harmonic linearization (Popov, 1973). In the presence of several essential nonlinearities, obtaining a relatively accurate solution becomes largely uncertain. The solution of such a system of linear ordinary differential Equations 9 – 15 can be obtained using the fourth-order Runge-Kutta method (Kamke, 1959). We write down the basic relations for the approximate integration of a first-order differential equation $dy/dt = f(x, y)$ with the initial condition $y(x_0) = y_0$ with step hr characterizing some interval, obtaining the current values of the variables $x_i = x_0 + ih_r$; $y_i = y(x_i)$, (i = 0, 1, 2, ...) (Equations 16 – 21).

It is believed that the task should be well conditioned (Kireev and Panteleev, 2015). If this condition is not met, small changes in the initial conditions can greatly distort the solution. The solution can be sought with constant steps h, and the error is estimated from the results of calculations. In some works, for example, using the Runge-Kutta-Meerson method, the error is estimated at each step and, depending on this, a decision is taken to change the step of the calculations. From this, it follows that calculation with the above-mentioned algorithm can be performed with changing coefficients at each step, but it is important to avoid the loss of stability of the computation.

Such an approach can be used to solve

differential equations with essential nonlinearities. In connection with the fact that in the considered system of Equations 9 – 15 there are several significant nonlinearities, it was decided after each of the four stages of calculating the variables y_i to make a correction with allowance for the parameters of each nonlinearity. Mathematical modeling of such a problem was successfully carried out using the original computer program as follows.

Taking into account the control actions, we write the system of equations of motion transfer in the hydraulic drive Equations 9 – 15 in the form Equations 22 – 27. Here Q_{pv}, Q_{dv} – flow rate of leakages through the safety valves, respectively, from the pressure and return pipes; K_c, a_1, a_2 – coefficients of the control mechanism; U – voltage on the windings of the control electromagnet. In order to implement mathematical modeling on a computer, we introduce the notation for each variable: $Y(1)$, $Y(2)$,…, $Y(6)$,, derivatives for these variables: $F(1)$, $F(2)$,…, $F(6)$, and also for each constant: c_1, c_2,…, c_n (Equations 28 – 51). Then the system of Equations 22 – 27 can be represented in the form Equations 52 – 58. In this case, the law of change of the moment of resistance was described by Equation 59. Here k is a certain number that characterizes the features of the generation of oscillations of the disturbing effect. An algorithm for calculating the system of differential Equations 52 – 58 is shown in Figure 4. Printing was carried out through 100 cycles of calculations.

As nonlinearities, introduced the pressure limitation in both lines is set to P = 15 MPa and the friction in the load and the hydraulic motor. In addition, in the counting process, a restriction on the rotation speed $\Omega_2 \geq 0$ was introduced. In Figure 5 shows the results of modeling the transient process in the hydraulic drive after a sudden change in the control voltage causing an abrupt increase in the flow rate of the pump Q from zero to 0,335Q. The following initial data are accepted: moment of inertia J = 0,05439 Nm*s^2 in the absence of a static load; length of lines l = 2m; coefficient = 19,6 1 / (Nmsek), the initial pressure in the filling shunt line P_0 = 0,7 MPa and the parameters of the control mechanism a_1 = 5,36*10^{-2} s; a_2 = 5,76*10^{-4}s^2; K = 0.02 1 / V.

The simulation was carried out with the help of a computer with the steps of counting H=10^{-4}; 2*10^{-4}s. The error did not exceed 10%. In the process of calculations, all the calculation results at which the curves were plotted were recorded in a special file.

The results of the full-scale experiment and numerical simulation turned out to be convergent (Kondratenko, 2005).

CONCLUSIONS:

The problem of mathematical modeling of dynamic processes in essentially nonlinear mechanical systems with lumped parameters is presented on the example of hydraulic drive functioning. A numerical solution of a system of differential equations with essential nonlinearities is obtained, obtained by the approximate fourth-order Runge-Kutta-Meerson integration method. The results of modeling the change in the speed of rotation of the output link and stresses in mechanical lines after a stepwise increase in the control action are given.

The proposed method of numerical simulation can be used to study the operation of any mechanical systems with lumped parameters of force lines containing several essential nonlinearities. Due to the fact that modern requirements for various devices require knowledge of not only static but dynamic properties, the consideration of dynamics will make it possible to determine not only the features of the movement of the device but also the fluctuations of stresses. The allowance for voltage fluctuations makes it possible to evaluate the resource of the mechanism, relying not on the coefficients obtained through statistical studies, but on identifying dangerous parts and details taking into account real amplitudes and frequencies.

Since in dynamic analysis it is very important to know the frequency characteristics of systems, the proposed method makes it possible to determine the response of any system to different input signals. By introducing harmonic signals with different frequencies, one can construct frequency characteristics of nonlinear systems.

Due to the fact that the count rate of the proposed digital method is significantly higher than using known programs, this method can be used to analyze the motion and make a decision about the direction of movement and its features in real time for fast moving objects.

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$$\frac{\partial\Omega}{\partial t}=\sum_{i=1}^{n}\frac{\partial\Omega}{\partial\zeta_i}f_i \quad (1)$$

$$f_i=\frac{d\zeta_i}{dt} \quad (2)$$

$$\frac{d\zeta_i}{dt}=f_i\left(t,\ \zeta_1,\ \zeta_{2,}\ ...,\ \zeta_n\right) \quad (3)$$

$$\frac{v\partial^2 u}{\partial t^2}-\frac{\partial\psi(u)}{\partial x}=Q(x,t) \quad (4)$$

$$\psi(u)=Ef\frac{\partial u}{\partial x} \quad (5)$$

$$u(x,t)=\sum_{k=1}^{\infty}H\theta(x)\sin(pt+\alpha) \quad (6)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\left(\frac{dq_i}{dt}\right)}\right)-\frac{\partial T}{\partial q_i}=Q_i \quad (7)$$

$$Q=-\frac{dU}{dq} \quad (8)$$

$$f_p\upsilon_{p1}(t)+Kp_{p1}(t)+w_h^2\vartheta_p\frac{dp_{p1}}{dt}=Q(t) \quad (9)$$

$$\vartheta_p=\frac{V_p}{\kappa w_p^2} \quad (10)$$

$$f_d\upsilon_{p2}(t)+Kp_{p2}(t)+w_h^2\vartheta_d\frac{dp_{p2}}{dt}=Q(t) \quad (11)$$

$$\vartheta_d=\frac{V_d}{\kappa w_h^2} \quad (12)$$

$$w_p\Omega_1(t)=f_p\upsilon_{h2}(t)-\tau w_h^2\left[p_{h1}(t)-p_{h2}(t)\right] \quad (13)$$

$$w_h\Omega_2(t)=f_d\upsilon_{p2}(t)-\tau w_h^2\left[p_{h1}(t)-p_{h2}(t)\right] \quad (14)$$

$$(1-c)w_h\left[p_{h1}(t)-p_{h2}(t)\right]=J\frac{d\Omega_2}{dt}+h\Omega_2(t)+M_r(t) \quad (15)$$

$$y_{i+1} = y_i + \Delta y_i \tag{16}$$

$$\Delta y_i = \frac{1}{6}\left(K_1^{(i)} + 2K_2^{(i)} + 2K_3^{(i)} + K_4^{(i)}\right) \tag{17}$$

$$K_1^{(i)} = h_r f\left(x_i, y_i\right) \tag{18}$$

$$K_2^{(i)} = h_r f\left(x_i + \frac{1}{2}h_r,\ \ y_i + \frac{1}{2}K_1^{(i)}\right) \tag{19}$$

$$K_3^{(i)} = h_r f\left(x_i + \frac{1}{2}h_r,\ \ y_i + \frac{1}{2}K_2^{(i)}\right) \tag{20}$$

$$K_4^{(i)} = h_r f\left(x_i + h_r,\ \ y_i + K_3^{(i)}\right) \tag{21}$$

$$\frac{d\Omega_2}{dt} = \frac{1}{J}\left[\left(1-c\right) w_h \Delta P\left(t\right) - M_r\left(t\right) - h\Omega_2\left(t\right)\right] \tag{22}$$

$$\frac{dP_p}{dt} = \vartheta_p \frac{C\left(1\right) - w_p\Omega_2\left(t\right) - Q_{pv}\left(t\right) + Q_{dv}\left(t\right) - \tau w_h^2 \Delta P\left(t\right) - 0{,}1\tau w_h^2 P_{h1}\left(t\right)}{V_p Z_1\left(\omega\right)} \tag{23}$$

$$\Delta P\left(t\right) = P_p\left(t\right) - P_d\left(t\right) \tag{24}$$

$$\frac{dP_d}{dt} = \vartheta_d \frac{w_h\Omega_2\left(t\right) - C\left(1\right) + Q_{pv}\left(t\right) - Q_{dv}\left(t\right) - \tau w_h^2 P_{h2}\left(t\right)}{V_d Z_1\left(\omega\right)} \tag{25}$$

$$\frac{d\Phi}{dt} = \frac{1}{a_2}\left[K_c U\left(t\right) - \frac{d\varphi_0}{dt} - \frac{d\varphi}{dt} - a_1\frac{d^2\varphi}{dt^2}\right]. \tag{26}$$

$$C\left(1\right) = \varphi_0 \Omega_1 w_p \tag{27}$$

$$Y\left(1\right) = \Omega_2\left(t\right) \tag{28}$$

$$Y\left(2\right) = P_p\left(t\right) \tag{29}$$

$$Y\left(3\right) = P_d\left(t\right) \tag{30}$$

$$Y\left(4\right) = \varphi_0\left(t_0\right) \tag{31}$$

$$Y\left(5\right) = \frac{d\varphi}{dt} \tag{32}$$

$$Y(6) = \frac{dY(5)}{dt} \tag{33}$$

$$F(1) = \frac{dY(1)}{dt}. \tag{34}$$

$$F(2) = \frac{dY(2)}{dt} \tag{35}$$

$$F(3) = \frac{dY(3)}{dt} \tag{36}$$

$$F(4) = \frac{dY(4)}{dt} \tag{37}$$

$$F(5) = Y(6) \tag{38}$$

$$F(6) = \frac{d\Phi}{dt} \tag{39}$$

$$c_1 = \frac{1}{J} \tag{40}$$

$$c_2 = (1 - c) \tag{41}$$

$$c_3 = w_h \tag{42}$$

$$c_4 = w_p \tag{43}$$

$$c_5 = \vartheta_d \tag{44}$$

$$c_6 = \vartheta_p \tag{45}$$

$$c_7 = \tau \tag{46}$$

$$c_8 = h \tag{47}$$

$$c_9 = V_p \tag{48}$$

$$c_{10} = V_d \tag{49}$$

$$c_{11} = a_1 \tag{50}$$

$$c_{12} = \frac{1}{a_2} \tag{51}$$

$$F(1)=c_1\left\{c_2c_3\left[Y(2)-Y(3)\right]-M_r(t)-c_8Y(1)\right\} \quad (52)$$

$$F(2)=\frac{c_6}{c_9}[C(1)-c_4Y(1)-Q_{pv}(t)+Q_{dv}(t)-c_7c_3^2[Y(2)-Y(3)]-0,1c_7c_3^2Y(2)] \quad (53)$$

$$F(3)=\frac{c_5}{c_{10}}\left[c_3Y(1)-C(1)+Q_{pv}(t)-Q_{dv}(t)-c_7c_3^2Y(2)\right] \quad (54)$$

$$F(4)=0 \quad (55)$$

$$F(5)=Y(6) \quad (56)$$

$$F(6)=c_{12}\left[K_cU(t)-Y(5)-c_{11}Y(6)\right] \quad (57)$$

$$C(1)=c_4\varphi_0\Omega_1 \quad (58)$$

$$M_r=M_0\left[1+0,005\sin(k\Omega t)\right]\text{, Nm} \quad (59)$$

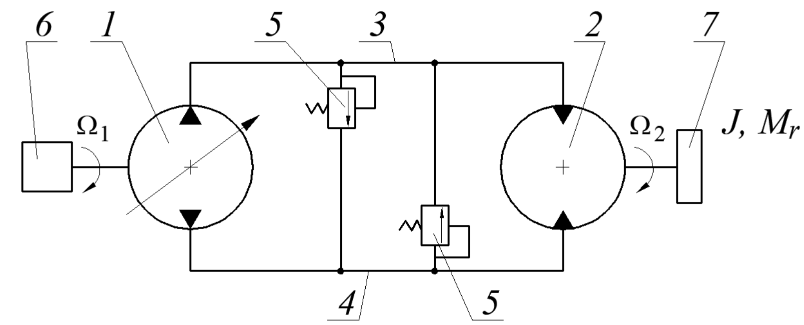

Figure 1. *Structural diagram of the volumetric hydraulic drive: 1 – adjustable pump; 2 – the hydraulic motor; 3 – pressure line; 4 – the transfer line (drain line); 5 – safety valve; 6-electric motor; 7 – the executive body*

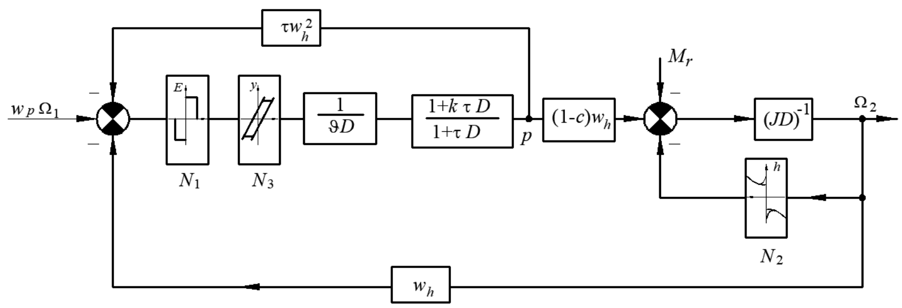

Figure 2. *Structural scheme of motion transfer:* N_1 *– non-linearity of the elastic modulus;* N_2 *– nonlinear friction in the load;* N_3 *– backlash at the input of the hydraulic drive;* N_4 *– restriction on pressure drop;*
τ *– relaxation constant;* κ *– coefficient of the ratio of the isothermal (*κ_1*) and adiabatic (*κ_2*) elasticity modulus of the working fluid,* $\kappa = \kappa_2 / \kappa_1$*; D – differential operator,* $D \equiv d/dt$*.*

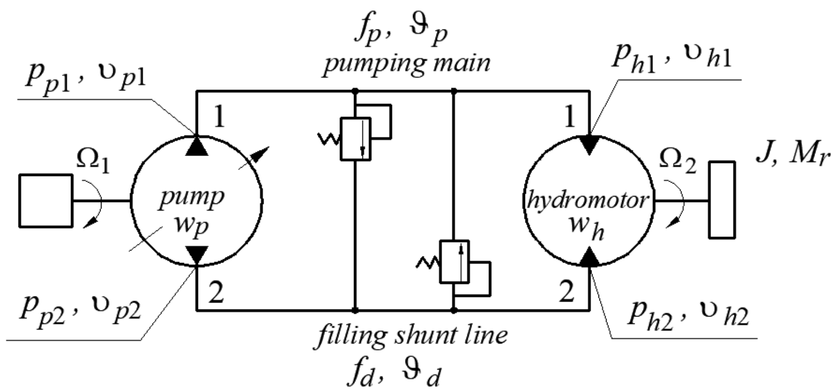

Figure 3. *The calculation scheme*

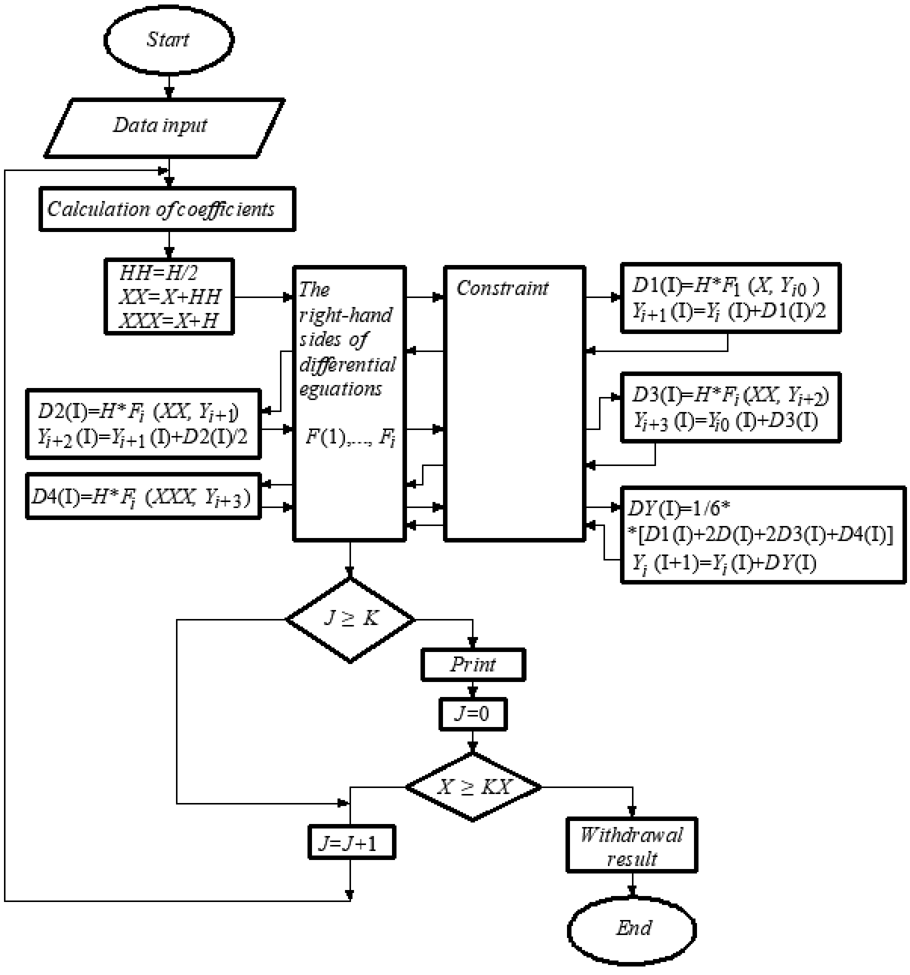

Figure 4. *Algorithm for calculating the system of differential equations*

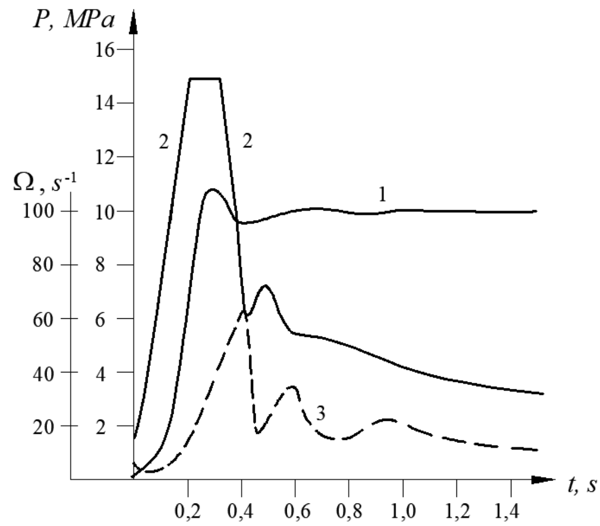

Figure 5. *Calculated oscillogram of the transient process in hydraulic drive: 1 – Ω – rotation speed of the motor shaft; 2 – P_p – pressure in a pressure main; 3- P_d – is the pressure in the supply (makeup) line*